MIGRATION, EDUCATION & ECONOMIC DEVELOPMENT
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Abstract

Migration and education have long been issues of major interest in regional science, economics and geography. The interest is quite understandable because they can have substantial consequences for society, individuals, regions and families. Exactly how migration and education affect economic development and inequality (positively and negatively) is not fully resolved. Recognising this, we look at the positive and normative roles that migration and education can play in determining economic prosperity. To serve this purpose, the thesis builds frameworks using dynamic general equilibrium theory to provide some analytical solutions and applies these empirically with panel data to determine the impacts of migration in a market economy. Overall, empirically we find that disparities among individuals and regions still exist despite the migration process. However, migration and education are both susceptible to market failure due to fixed costs and liquidity constraints. The thesis moves on to examine the role and incentives for facilitating education and migration in non-market environments. In particular, we examine these processes within intergenerational family settings. Intra-family intergenerational transfers motivated by altruism are studied but we show that public intervention is also generally necessary to achieve Pareto optimality. Finally we consider the empirical evidence on intergenerational mobility in education and occupation for migrants.
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1 Introduction

Education and migration are important factors for economic development. Migration and economic development are closely related to one another: economic development motivates migration, and migration influences economic development in turn. Migrants normally coming from relatively low income areas with different education level move to relatively high income places in order to increase their individual welfare. This movement influences the economic development of both migration sending and receiving areas. The impacts can be either positive or negative. Through migration, individuals gain a high level of income. Some of the income can be sent home as remittances, which can be invested in education or household production, in ways that create new income opportunities at home. However, the remittances can be also spent on daily consumption, generating no long run effects on economic development. For the destination, those of the migrants who are highly educated move with high levels of human capital and increase the productivity in host countries, which could further boost economic development. The migrants can also be low skilled, competing with local low skilled residents who could be easily unemployed. The unemployed natives would claim a welfare subsidy from the state and generate zero productivity. This has a negative effect on economic development. Similarly, in every sense, education is one of the fundamental elements of economic development. Education enriches individuals’ human capital and productivity levels. It improves their standard of living and generates social benefits to families and society, benefiting economic development. In addition, it affects the income distribution and secures economic progress. Controlling for appropriate different characteristics among people, the distribution of education matters for the distribution of income. A highly educated person is more likely to stay in the upper part of the income distribution. A low educated individual has less chance to move to the top part of the income distribution. In this sense, education matters for distribution and inequality. Overall, it is very likely that migration and education are integral parts of economic development in all countries even though migration and education do not always have positive effects and may not be panaceas for economic development.

Apart from the facts stated above, economic inequality is one important measure for economic development from a micro aspect (e.g. distribution and an absolute poverty line). The impacts of migration and education on economic inequality is also important to deepen our understanding of economic development. For instance, migration and education can positively affect the economic development of a place but may not change the distribution of income of that place. Everyone can be made better off, meaning the whole distribution of income is shifted to the right. But the rank of each individual can stay the same. The inequality level stays the same. Unless the whole distribution shifts above the poverty line, some poor people still cannot move out of poverty. Therefore, regarding the distribution of individual welfare, economic inequality and poverty are essential dimensions for economic development.
1.1 Education, migration and economic development

In this part, we briefly browse the meaning of economic development, links between migration/education and economic development. From our point of view, economic development is the development of economic wealth of countries, regions or communities for the well-being of their residents. The well-being covers a range of things such as human capital level, health condition, life expectancy, income/wage, living conditions, equal opportunities etc. Some well-being relates to how countries, regions or communities can advance their economies, namely economic growth. For example, economic growth benefits human capital development since economic growth is more likely to lead individuals and families to increase expenditure on education and health, which furthers economic development. The increased human capital level and the health condition further contribute to economic growth. This confirms a two-way relationship between economic growth and economic development and one is important for the other.

In the following part, we study the relationship between migration and economic development linking to economic growth.

International migration is an important component of globalization and economic development in many less developed countries and some developed countries. The number of international migrants has increased over the past 40 years, approximately from 76 million (about 2% of world population) in 1965 to 188 million (about 3% of world population) in 2005 (Fig 1). International migration could be either a positive or a negative factor for economic development for both destinations and origins (Taylor, 2006; Borjas, 1996). (i) Migrants raise economic development in origins through remittances, the income that migrants sent home. For instance, remittances account for 11% of the gross GDP of Guatemala, 16% of the total GDP of El Salvador (Taylor, 2006). International migration plays a positive role in the economic development of some developing countries. (Felbermayr et al, 2008) use cross-section data of several countries and find a robust and non-negative effect of immigration on real per capita income in the origin by using geography-based IVs and controlling for differences in institutional quality, trade, and financial openness. A 10% increase in the migrant stock leads to a per capita income gain of 2.2%. (ii) International migration also has a positive effect on education in the origin. The volume of migrants and the amount of remittances are positively correlated with the secondary school enrollment rate in the Philippines (Theoharides, 2013). She interprets the increasing schooling enrollment rate partly as a result of remittances. Some amount of the remittances are spent on educational investment. This increases the school enrollment rate. (iii) Other studies find that the migrant sending countries suffer from low levels of human capital (brain drain) and no long run effect of remittances on economic development. Among 188 million migrants, there are millions of highly educated people who moved out from developing countries to developed countries (Raveesh, 2013) and
some of the remittances can be spent on daily consumption but not on long run economic development such as agricultural production (European Parliament, 2014). (iv) Meanwhile Borjas (1996, 2000) claims that the impact of international immigration on the host country depends on how the skill distribution of immigrants compares to the skill distribution of the native-born population, suggesting that international migration has no big negative effect but a positive impact on economic development in the US when the immigrants and natives are complementary and the impact is negative when the immigrants and natives are substitutes.

Recently, internal migration in general and rural-urban migration in particular have become more popular in the economic development literature. Internal migration in China is one of the most extensive and eye catching in the world. Since 1979, "China’s urban population has grown by about 440 million to 622 million in 2009 and of the 440 million increase, about 340 million was attributable to net migration (although some are due to urban reclassification)" (Chan, 2010). The volume of migration in such a short period is likely the largest in human history. Following this, we select China to look at the impacts of internal migration on economic development. Figure 2 suggests that the volume of internal migrants is increasing over time and the growth rate is increasing as well. The coast regions such as Guangdong (30% of the total internal migrants move to Guangdong) and Zhejiang (15% of the total internal migrants move to Zhejiang) are the two regions attracting largest internal migrants in China. Most of the migrants come from western and central regions, such as Sichuan and Anhui, ranked as the two highest number of emigrants in 2002 (the darkest color refers to the region with the highest volume of emigrants, see Fig 3). Even though China has this substantial number of internal migrants, there is an institutional moving barrier, namely the Hukou. The observed number of migrants might be far less than the real number if the Chinese government relaxed the Hukou system. The hukou barrier between less developed provinces and well developed provinces still exists.
Ideally, in order to empirically understand the effects of this institutional moving barrier on internal migration in China, we would look at the empirical data on how Hukous are acquired. Unfortunately, there is no data publicly available. Nevertheless in the late 1980s, many local governments started some form of "urban citizenship for sale" practice to get revenue, even though the central government opposed this practice and this policy had been abolished a while ago. A well-respected researcher in China, Han Jun, defines broadly the rates of Hukou in 1994 (based partly on administrative knowledge on Hukou value and partly on oral communication with people who purchased Hukou) as Table 1 (cited in Chan & Zhang, 1999). If we apply Hu Jun’s hukou rates of the different administrative units to different provinces, the hukou rates of Beijing and Shanghai will be higher (about 3 or 4 times) than that of Sichuan, Anhui, Hubei and Henan (Chan & Hu, 2003). Through Table 1, we could get a rough idea of the moving barriers and acknowledge there is an institutional moving barrier on internal migration in China. Knowing that there are some repressed internal migrants, migration data does not reveal potential movements and if we want to look at the impact of migration on economic development, it would be better to turn for help to simulation. In a recent study, the World Bank (2005) simulates the impacts of reallocation from low-productivity agriculture sectors to high productivity sectors on Chinese economic growth. It reports that China would gain 6.4% increase in GDP growth after moving 10% of labour out of the agriculture sectors. With this 10% labour movement, the western and central region would gain by 8.2 and 5.7 percent in GDP growth. As we mentioned above, economic growth can improve human capital development, which furthers economic development.

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1The hukou rates are positively correlated with the administrative status of a place. The hukou value of metropolitan areas such as Beijing and Shanghai is much higher than less developed areas. This reasonably explains why the number of immigrants in Beijing is very low. Based on the definition of urban areas in the 1990 Census, all districts are divided into 4 administrative units: provincial-level units; prefectural-level units; county-level units; township-level units. And the villages are administrated by township-level administration. According to the value of the administrative status of a place and the districts rank, the rate of village hukou has the lowest rate compared with that of county hukou and that of town hukou.
Education is fundamental to economic development. On average, there is a positive link between education and income (Fig 4, web source\textsuperscript{2}) over 1975-2005 with respect to different educational levels. There is an upward time trend for educational return over 1975-2005, suggesting that the average return of education is increasing. The more an individual is educated, the higher the average income he can get. The average income gap between advanced education and low education (e.g. high school) is increasing, saying that education becomes more and more crucial for income distribution (income inequality) over time. Besides the private return, education also generates some social returns. We use a report from the World Bank (Psacharopoulos and Patrinos, 2002) to look at the social and private return to another year of schooling. They measure the private returns by the coefficient of individual years of schooling in an income-education regression and the external returns by the coefficient of average years of schooling in a relevant geographical area. The average years of schooling in an area can be a proxy for positive externalities (e.g. the benefits of private education investment may generate positive benefits to family, friends and society; a region with a high level of average education can provide positive externality to people; people learn from each other). Table 2 shows on average, the private returns to another year of schooling are higher than the external returns. There are both positive private and external gains from education but the

\begin{table}[h]
\centering
\begin{tabular}{|l|c|}
\hline
Prefectural-level cities & More than 10,000 yuan \\
\hline
County-level cities & 5,000-10,000 yuan \\
\hline
Cities and towns below county-level & 2,000-5000 yuan \\
\hline
\end{tabular}
\caption{Hukou value}
\end{table}

\textsuperscript{2}Web source: http://www.outsidethebeltway.com/americans-under-educated/
private gain is the larger. Nevertheless, the private returns are positively related to social returns. In addition, the returns to primary education investment are higher than the returns to higher education investment. It suggests that education investment in primary education especially for developing countries is very important.

1.2 Migration, education and economic inequality

Migration and education are similarly important for economic inequality. The importance of inequality comes from ideas of welfare economics and moral philosophy (Rawls, 1971; Sen, 1999). From a welfare economics perspective, migration and education are two important development tools for economic development. Through migration, individuals can move from low income regions to high income regions and tend to get high incomes, increasing their individual welfare and escape from inequality. However, the chance to escape from inequality might depend on the existence of equal opportunities. An individual or a household cannot move out of inequality through migration when there is a high moving cost and the individual/household is liquidity constrained. Through education, individuals can increase their human capital level and are more likely to gain high incomes, improving their living standards.
But when the tuition fee is too high, then only the rich people can go to university not the poor ones. From a moral philosophy perspective, Rawls (1971) and Sen (1999) both believe unequal opportunities are one of the important factors generating economic inequality. Along the line of Rawls’ argument, each individual should have equal opportunity to access the resources (economic and environmental) to meet their basic needs. Individuals have the freedom to migrate and have the equal opportunities to receive education to gain social justice. Consistent with Sen’s idea, we recognise the existence of the inequality and consider that it can arise from a form of capability deprivation (e.g. different genetic abilities). To prevent such inequality, we believe it is important for individuals to have liberty, freedom, social opportunities, protective security. People with different abilities are entitled with equal opportunity to go to school and to migrate to escape from inequality.

Migration and education are important for economic inequality in both ways (reduce and enlarge economic inequality). Equal opportunities could give support to migration and education in eliminating economic inequality. (i) The economic inequality can be due to economic environment such as family wealth. The family wealth is an important factor for inequality. Some are born in rich families and some are not. Those rich family children might start from better positions and can more easily get access to better resources, such as better private schools. When there exists equal opportunities and no institutional barriers, the economic inequality could be eliminated through migration and education. Under a society with free migration and no institutional barriers (e.g. Hukou in China), the poor members can be compensated through migration, moving away from poor regions to rich regions to get high income to raise individual wealth. With equal genetic factors, if both poor and rich children can go to the same school and receive the same quality of teaching service, then everyone could start from the same initial line. When there is a big institutional barrier or a market discrimination or a liquidity constraint problem, then the effect of migration and education on economic inequality might not be that significant. If there is a big entry fee for migrants or different tuition fees for different education services/quality, then only rich people without liquidity constraints can migrate from poor regions and only rich children can go to better schools. (ii) The economic inequality can also be caused by genetic characteristics. Individuals are born with different genetic traits. Genetic differences can due to inherited traits, patterns or characteristics were handed down from parents to offspring or due to random shocks (e.g. disabilities). For those who have genetic diseases or disabilities or are mentally retarded, normal education or normal health care is not enough for them. They might need extra resources or special education to offset their disabilities to gain the same well-being as healthy people. In this case, equal opportunities become extremely important in helping reduce inequality. (iii) Thirdly, the economic inequality can also be caused by luck or any random shock. We live in such a complex society, we have limited knowledge for future outcomes. Everyone
faces idiosyncratic shocks. Those shocks alter our wealth, health which can affect the well-being of others living around us, which further lead to economic inequality. A random health shock may negatively affect an individual’s productivity in rural production, which further reduces his income in rural area. Migration can be an insurance tool to diversify the shocks. The low rural income can be offset by the high urban income earned by the migrants, making the total family income stay at the same level as other rural families who do not receive bad health shocks. Meanwhile, a low urban income caused by bad luck or low employment probabilities or economic shocks in urban areas can be offset by the rural income. Besides migration, education can be a useful tool to deal with uncertainties but might enlarge inequality sometimes. Individuals can use education or a high level of human capital to ensure the success rates of job seeking and high employment probabilities. When the economy is hit by a bad shock, firms may want to reduce the number of employees to secure the profit. The highly educated ones or those with special skills are more likely to keep their jobs and incomes. In this case, the income gaps between employed and unemployed might be enlarged.

Having stated the concepts and controversial effects above, we will look at some empirical evidence to understand how migration and education affect economic inequality.

The effects of migration on economic inequality vary with liquidity constraints, family wealth, the use of remittances and the skill distributions of migrants and natives. Some find that inequality goes up when migration flows go up, and others find the opposite. For the origin, Taylor (2006) argues that migration normally tends to come from households at the upper-middle part of the income distribution. Those households are less liquidity constrained, more wealthy, have a bigger size of networks and are slightly more educated. Through migration, they send remittances back home. The positive remittances only yield benefits to these upper-middle households, whilst the households at the bottom part of the income distribution cannot afford to move because of liquidity constraints or a small size of networks. Then the income inequality gets wider through migration. However, if eventually the poor households get some positive spillover, then in the long run in an economy with a large emigration the income inequality will be narrowed. E.g. the migrants from upper-middle household return to the origin with a large size of capital inflows and set up local factories. The poor households can start to build their wealth and human capital endowment by working for these local factories and resolve the liquidity problem. Over time, the poor household members can migrate and the income inequality gets smaller. Figure 5 shows the link between remittances sent back by migrants and income inequality. Income inequality goes up when only a small percentage of households have migrated abroad but goes down when more and more people migrate abroad. On the other hand, remittances do not necessarily lead to long-term investment but only short-term investment or consumption.
(food, health, household’s need). For example, a study done by Zhu et al (2009) finds that remittances sent by rural-urban migrants are not spent on agriculture production but more on daily consumption, elderly health care and education. The elderly health care could not generate economic growth for the rural economy as the old do not participate in rural agricultural production anymore. The rural economy does not benefit from education investment either because educated individuals generally migrate away from the rural area and contribute to the urban economy. Even though they will send remittances back, the money would be spent on daily consumption. The rural economy won’t benefit from this in the long run. Besides, whether an investment qualifies as productive or not depends on the sociocultural and economic considerations of each country. But sometimes migrants are celebrated for their key role as promoters of changes in shaping socioeconomic and political reform in home countries. They might bring large capital inflows and create many new jobs for local rural people, generating positive economic growth and increasing the incomes of local rural people, then migration can shape the rural economy in the end. For the destination, the impact of immigration on the economic well-being of the native population of the receiving place/region/country is less obvious and the empirical studies are less widely done, even though there are a lot of studies on the impacts of migration on local labour market in host countries. As far as we know, there are not so many studies on the impact of migration on economic inequality. A general argument from Borjas (1996, 1999) Chiswick (2005) says that "immigrants do not have a common effect on the native population; the effect depends on the relative skill characteristics and property rights of the immigrants and natives, although immigrant workers tend to raise the overall income of the native population; skilled immigrants tend to raise the level of income of the native population, reduce income inequality and are not likely to be substantial recipients of income transfers; unskilled immigrants tend to increase income inequality in host country" (they promote productivity insignificantly and compete with local residents, leading to increasing unemployment); the immigrants moving with wealth tend to create more jobs in the destination, to generate more working opportunities and to reduce income inequality in destination. We will study the impacts of migration on economic inequality in destination both in this thesis and future studies.

Education level does matter for income distribution. Abdullah et al (2011) run a simple regression (inequality: Gini coefficient/income share of the top quintile/income share of the middle quintile/income share of the bottom quintile/Theil index etc.) on education (different levels of school enrollment or attainment) using a meta-data set (covering nearly all countries such as US, African, European Union, Developing countries etc.) for 1960-2006. They find that education decreases the income share of the top class and raises the share of the bottom class and has no impact on the middle earners. Inequality in education does matter for income inequality. If the inequality in
education is very big, then the inequality in income could be large as well. Meanwhile, education is one channel to remove the inequality and to provide people with equal opportunities, which further affects income inequality. Gregorio & Lee (2002) study the relation between income distribution and educational attainment using panel data covering 1960-1990 and 49 countries. They estimate the Gini coefficient of income (a measure of the income distribution) on the standard deviation of schooling in the population for a given year, government social expenditure etc. They find empirically that countries with higher educational levels or less dispersion of educational attainments among the population, or with higher levels of economic development, tend to have a more equal or less unequal income distribution and present some graphical examples for the link (we list of them here, see Fig 6). They also find a positive contribution of government social expenditure to a more equal income distribution. They suggest policy intervention such as expanding public investment on education could help reduce income inequality. This public intervention can be interpreted as equal opportunities, helping poor children to enter the same schools as rich children to reduce dispersion of educational attainments among the population, which further reduces income inequality.

1.3 Limits to scope of migration and education

Migration and education both involve up-front costs so are inaccessible to the poor if there are imperfect capital markets. Does this need policy intervention? Is there any tool to overcome the liquidity constraints in a decentralised system? In this part, we propose two tools to overcome the liquidity constraints in a decentralised system.

Altruism and transfers are two important tools for economic development. Conceptually, in a decentralised system without any access to capital markets, the relatively wealthy and altruistic working generation can finance
the education of the young generation who is at the initial level of his lifecycle wealth profile, and due to lack of collateral, cannot get access to capital markets. Through this, the young generation can receive education and build their human capital without being constrained by liquidity. This working generation could get repayment or receive transfers from the young generation when the young generation start to work. Practically, we present one study to look at the determinants of altruistic behaviour and transfers. Due to data limitations in altruism, we use the results from a theoretical model to show the importance (Rapoport & Vidal, 2007). Rapoport and Vidal (2007) develop a simple general equilibrium model of endogenous intergenerational altruism and growth. Starting from some initial family wealth (inherited bequest), one-period lived and altruistic parents allocate the wealth between bequest to children, their own consumption and child-oriented expenditures. In a closed economy, parents work for a representative firm and the bequests between generations are the main economic input for the firm. Altruistic transfers have an opportunity cost for parents. Therefore, the altruistic behavior are directly affected by the economic conditions such as family wealth. In turn, altruism affects the economic development and growth. Intergenerational transfers are determined by the degree of altruism and the economic growth rate at earlier stages.

1.4 Research questions of thesis

Having stated the importance of migration and education on economic development and inequality as well as some limitations, we raise a few research questions to guide the whole thesis.

1. Suppose there are income disparities among regions, is migration a good channel for individual economic development (in particular to improve individuals’ well being status)? Can migration can help people escape from income inequality?
2. If there are moving barriers such as a fixed moving cost and Hukou, how do costs affect the economic development process and migration choices? Further, how do costs affect income inequality?

3. Apart from the economic inequality, we suppose there are other social disparities (e.g. infrastructure, public good provision, urbanization levels) among regions. Is migration still a good channel for individual economic development? Can migration help people escape from economic and social inequality?

4. In a decentralised economy, what are the impacts of education on economic development and inequality?

5. Can migration and education help increase intergenerational mobility? What are the correlations between mobility and inequality?

We will try our best to answer those questions in four chapters.

Chapter 2 tries to answer questions 1 and 2. It considers income differences between two regions, developed and undeveloped ones. Linking to question 1, migration can help people improve individuals' well being status and escape from inequality. Through the migration without moving cost, people can move from poor regions to rich regions to gain high income levels to improve well being and escape from poor regions. However, each coin has two sides. In a dynamic system, large immigration flows can reduce the income disparities but perversely may also increase them. When a region receives too many immigrants, the job market shows excess of labour supply causing high unemployment rates and falling wage rate. This increases gaps between employed and unemployed workers within as well as between regions. Migration affects economic development and inequality in a negative way. This leads us to look at the two way causal links between migration and development/inequality to fully understand the impacts. In addition, if we consider moving cost relevant to question 2, migration has the same two-sided effects but not everyone can improve the well-being through movement. Those who are less liquidity constrained are able to move. Those who are liquidity constrained get stuck in poor regions. Moving cost may enlarge income inequality and prevent people from having the same development opportunities.

Chapter 3 tries to answer question 3. It concedes that there are differences in other aspects mentioned above but also extends two location choices to multiple location choices. It tries to answer whether migration can still play the same role in this context. To do this, this chapter develops a structural micro-founded model of aggregate net migration flows to study how migrants choose between multiple locations using multiple criteria. Most migration models either do not handle multiple criteria and locations or lack micro foundation. We develop a model combining those two. By doing this, we can combine and decompose all the pull and push factors and look at how migration flows react to each of these. Each location can have different dominating push and pull factors, which attract
immigrants and push people to emigrate. Over time, we can look at what are the determinants for migration. Further, the labour demand between places can be different. The places with labour demand can continuously attract large immigration flows while others push people away. At the most attractive location, there are large immigration flows from other locations. At the least attractive location, there are large emigration flows. During this process, we will study whether the disparities between regions have been reduced during migration.

Chapter 4 tries answer question 4. In this chapter, we allow for a non-cooperative family environment without state intervention and capital markets (the young generation cannot get loans without having collateral) but add one coordination possibility: altruism, to study the impacts of education and transfers on development and inequality. We believe that individual well being can be improved and inequality can be reduced through human capital investment and family transfers. People gain a high level of human capital and enter the labour market to get high income, which increases well-being. The altruism assumption makes the relatively rich individuals transfer some money to the relatively poor individuals to equalize wealth. For instance, the young generation normally is more liquidity constrained. With the transfer, they can do human capital investment to have the chance to gain high income. This framework within household decision theory helps us understand the dynamics links between intergenerational transfers and education investment. It also helps us investigate whether family coordination can generate a social optimum without the help from government intervention. If not, then we will introduce a more comprehensive altruism system and/or a government instrument (tax/subsidy) to correct the failure.

Chapter 5 tries to answer question 5. It looks at intergenerational mobility in education and migration for rural-urban migrants in a developing country, China. Rural-urban migrants is a very interesting group, who witness the change both in rural and urban areas. Most of their parents still live in rural area and some of their children start to live in urban area. By comparing the education levels and occupation categories between migrants’ parents, migrants themselves and migrants’ children, we can see the changes over time. This helps us understand intergenerational mobility. Migrants can change their social categories through migrating from rural to urban areas. Their children can also change social categories by being taken by their parents and receiving education in urban areas. In addition, we apply Markov chain theory to study the distribution changes over time to better understand inequality. We suspect the inequalities can be reduced through increasing mobilities.

Along the line of those four chapters, the final chapter will conclude and propose a list of related new research problems.
Three musketeers: A dynamic model of capital inflow (FDI), the real wage rate and the net migration flow with empirical application

2.1 Introduction

The aim of this chapter is to study the income disparities among regions, the dynamic interactions between capital flow, the real wage rate and migration, and tries to answer questions 1 and 2 of Chapter 1. In a utility based maximisation process, people choose to move when the expected wage rate is higher than the sum of the current wage rate and the moving cost. Obviously, people cannot know exactly the employment probabilities at the moment when they choose to move. They only have some information on the distribution of labour demand and supply among regions. Therefore, in this chapter, we use a ratio of observed labour demand and supply to approximate the expected employment probabilities. Then, labour demand and supply are both important factors for migration decisions. Meanwhile, the migration flows inversely influence the real wage rate through labour supply. Given a fixed amount of jobs, large immigration flows are negatively correlated with employment probabilities which further discourage immigration. On the other hand, an increasing labour demand can offset the rising labour supply. In a standard production process, labour and capital are two main inputs. A positive capital inflow can generate more jobs and can increase labour demand when they are complements. Conversely, this can also be a negative factor for labour demand when capital and labour are substitutes. Thus, we need to consider the whole complex system allowing for the interactions between labour and capital markets and migration.

For this purpose, we provide an analytical framework to study the simultaneous interactions between capital inflow (FDI), the real wage rate and net migration flow into a region and apply this structural model with numerical calibration to Guangdong, a fast growing Chinese province with the highest net migration in the emerging world (UN, 2011). Except for Federici and Giannetti (2010) who build a dynamic model with these three endogenous variables to study complementarity between FDI and temporary migration in a one way (return) migration setting, the simultaneous interactions with two way migration and immobility have not been studied. The existing migration literature only refers to two dimensional interactions between capital and migration or between the wage rate and migration. In fact migration, wage rates and capital flows are interconnected through the labour and capital markets. In an immigration context, the positive link between immigration and capital inflow (FDI) has been confirmed empirically (Clark and Gertler, 1983; Buch et al., 2006; Foad, 2012; Ivlevs, 2006). In a return migration context, contrary to the "brain drain" effect of emigration on migrant sending countries (Docquier and Marfouk, 2005; Borjas, 2005; Kapur and Muhale, 2009; Commander et al., 2003), a positive and complementary dynamic link between FDI and labor mobility has been demonstrated by a group of return migration research scholars (Kugler
and Rapoport, 2007; Dustmann and Weiss, 2007; Dustmann and Kirchkamp, 2002; Ma, 2002). A related question is the interaction between migration and wages but the findings are mixed: Borjas (2003) argues that immigration has substantial impacts on wages in the host country, but Ottaviano and Peri (2007; 2008) and Amuri et. al. (2010) find small effects.

We develop a time continuous dynamic model of a system of piecewise differential equations. A key migration determinant is the expected income level which depends on the level of wages and the unemployment rate (which determines the chance of finding a job) in different locations. In turn these are determined by the technology, capital and labour endowments and the heterogeneous functioning of the labour market in different locations. But labour migration has a cost. Together these imply that the dynamics of immigration and emigration flows will respond differently to an expected wage gap between the two locations, and also there will be combinations of labour, wage and capital endowments under which labour movement does not occur due to the fixed migration cost. We start from a benchmark model in the short run\(^3\), in which the labour demand is determined by the representative firm’s profit maximization process with fixed wages and capital stock, net migration responds to expected labour income differences between host and origin locations. In this context, only migration adjusts and at any time there may be immigration, immobility or return migration. We then extend the time period to the medium run in which the real wage is flexible and adjusts according to the excess demand for labour in the host country but capital is still fixed. Finally we allow capital to adjust dynamically (through FDI), following the gap between the marginal product of capital in the host country and the world interest rate at time \(t\). We compare the equilibria and stability of these three cases. Empirically, we apply a general model (three flexible variables) with calibrated Cobb-Douglas production functions and estimate the dynamic adjustment speeds of wage rates, migration and FDI in 16 regions of Guangdong over 1990-2010. We claim three contributions in the chapter. No other study includes these simultaneous interactions which fills a gap in the literature, our chapter is the first to recognize the inherent regime shifts due to migration costs, the chance of getting a job and two way migration. We find that the effect of the elasticity of labour demand is an important factor in the local stability conditions. The framework predicts time series properties of real wage rates, migration and capital flows. Generally all may have cycles which will be of varying duration and so not in phase. Migration flows should show relatively flat peaks and troughs whilst real wage rates turn around more quickly. The correlation and cross auto-correlation between the series should vary with the phases of the cycle. In a time series context, this may be of econometric interest. The empirical results indicate that regions in Guangdong are heterogeneous but with positive simultaneous interactions between the three

\(^3\)Primarily for clarity of exposition, we build up the three dimensional dynamic system in stages.
endogenous variables. Some policy implications and further research directions are also suggested.

The chapter is structured as follows. Section 2 reviews the existing literature and frames this chapter’s contributions within those literatures. Section 3 presents the model in detail. Section 4 applies the general model with calibrated Cobb-Douglas production, describes the data and the empirical methodology. The results will also be discussed in this part. In the final section, we draw conclusions and some policy implications and further research direction are also suggested.

2.2 Review of the literature

2.2.1 Capital (FDI) and immigration

The existing empirical evidence on immigration and FDI concentrates on the empirical relationship between the two. Clark and Gertler (1983) use a time-series framework for 15 U.S. states over 1958-1975 to explore the relationship between immigration and capital. The close and positive similarities in the magnitude and the timing of fluctuations of immigration and capital growth for all states are indicated by their empirical work. More recent empirical work has also confirmed the positive interaction between capital and immigration, Barrry (2003) and Groznik (2003) show that immigration tends to lead to capital inflows to Germany. UK FDI favors destinations that also attract a large number of immigrants (Clemens and Williamson, 2000). Buch et al. (2006) use Tobit fixed effects panel regressions for 16 German states over 1991-2002 to analyze the relationship between FDI and migration flows empirically. They find that immigration and FDI are positively related, though the effect is largest for FDI from high-income origins. Complementary to the findings by Buch, Foad (2012) looks at the links between flows at the regional level for the 50 US states. The results in his paper strongly support a positive/complementary relationship between cross-border flows of immigration and FDI in US.

Similar to Foad’s (2012) paper, our empirical work studies the sign of the link by also looking at the regional level. A regional level analysis allows us to strip away any variation at national levels such as exchange rates. So FDI flows from the same source country to different regions in a single destination country will be subject to the same exchange rate. This approach helps hold any determinants at the national level constant and any variation in immigration must be due to be regional differences in FDI or vice versa. But the shortcomings of Foad’s paper are the measurement of FDI and the time frequency of the immigration data. Due to data limitations, he defines FDI as the number of majority-owned affiliates instead of computing the actual value of FDI. An obvious bias arises if a variable number of small enterprises enter the local market. Another disadvantage of his paper is that the immigration data are only updated every ten years while the FDI data are available on an annual basis. Annual migration flows cannot be explained. To overcome these flaws, our chapter estimates the sign of the link
by using annual migration data and the reported value of FDI at the regional level. In addition, our chapter also provides a structural theoretical framework to guide the empirical application. This predicts that there will be an agglomeration effect: regions with a high initial value of FDI and migration will tend to grow fastest. A region starting out with a high level of FDI or a large number of immigrants attracts more successive FDI and immigrants.

2.2.2 Capital (FDI) and return migration

Much research in economics is devoted to studying the impacts of immigration on the host economy (Borjas, 1989; 1994), but the beneficial aspects of migration for the sending country have received less attention. Contrary to the "brain drain" effect on native countries (Docquier and Marfouk, 2005; Borjas, 1989; Kapur and Muhale, 2009; Commander et al., 2003), migration can also be subsequently welfare-enhancing for those left behind, especially if capital-rich return migrants engage in entrepreneurial activities (or self-employed activities), undertaking capital investment in their home countries. The capital investment helps to overcome capital constraints and supports the economic development of the migrant’s home region (Dustmann and Kirchkamp, 2002). In addition, the role of migrant networks and the relatively high skill level of return migrants ensures that return migrants will be fully employed and this return migration will prompt FDI inflow to the native countries. The link between migration and FDI is supported both from a static standpoint and from a dynamic perspective. The standard static trade models (based on international factor endowment differentials) normally support the negative link argument while Kugler and Rapoport (2006) confirm the positive argument dynamically. To our knowledge, the only relevant theoretical framework is developed by Dustmann and Kirchkamp (2002), who are motivated by the empirical evidence from Turkish migrants (51.10% of Turkish return migrants become self-employed and do capital investment in Turkey, 6.2% choose to be salaried workers and 43.72% choose to retire) and develop a simple theoretical model to study the occupation choices of return migrants and the optimal immigration duration. Our chapter extends the empirical literature on the link between FDI and return migration and develops a wider theoretical framework to understand the connections between variations in FDI and return migration.

2.2.3 Immigration and wage

The findings of the interaction between immigration and the wage rate vary a lot. Brucker and Jahn (2011) apply Layard’s (2005) wage-setting approach (the wage rate reacts to the change in labor supply especially migration) to analyze the labor market effects of immigration into Germany and find that the effects are moderate (1% increase in immigration increases the unemployment rate by less than 0.1% and reduces the wage rates by 0.1%). By contrast Borjas (2003) indicates that immigration lowers the wage rate of competing workers: a 10% increase in immigration
reduces the wage rate of the natives by 3% – 4%. Especially, a significant negative effect of immigration on the wage rate of less educated natives is emphasized by Borjas (2003). Contrary to Borjas’s results, Ottaviano and Peri (2007) find small positive effects on the wages of highly educated and small negative effects on the less educated. Also, Ottaviano and Peri (2008) find that immigration has a small negative effect on the native average wage rate in the short run but a small positive effect in the long run.

The structural model Brucker and Jahn (2011) used to study the effects only refers to two dimensional interactions (the wage rate and migration). In their structural model, the wage rate is a function of labor and fixed capital. Given the market is competitive and the supply of native labor is inelastic, the wage rate equals the marginal product of labor when firm’s labor demand is equal to market labor supply. Similarly, in Borjas’s (2003) paper, capital is also assumed to be fixed. This assumption supports the argument "the labor demand is downward sloping" but rules out the possibility that the demand curve can be shifted "when the supplies of other imperfectly substitutable factors change" (Ottaviano and Peri, 2008). By enriching Borajs’s methodology and refining previous estimates, Ottaviano and Peri (2008) allow capital to be flexible to estimate the effects of immigration on the wage rate at the national US level. And they find an effect of immigration (a 10% increase in immigration) on the wage rate equal to −3.2% with no capital adjustment but −0.6% with capital adjustment. This result suggests that capital adjustment does matter for the effects of immigration on the wage rate. Our framework is in line with this result but goes further in studying the full interactions between capital, wage rates and migration. In detail, our chapter relaxes the fixed capital assumption, puts the three dimensional interactions in a dynamic framework and considers regime shifts (due to a fixed moving cost) between immigration, return migration and immobility in the migration dynamic equation. Moreover, instead of looking at the national level (different regions/areas within a nation share the same dynamic adjustment speeds), our empirical work uses an "area approach" (Card and Lewis, 2007) to estimate the dynamic adjustment speeds of capital, migration and the real wage rate for each area/region. If adjustment speeds are heterogenous between cities, assuming a common δ is a mis-specification. The "area approach" which considers the dynamic interactions at a more micro level allows us to test for common speeds. For example, cities can have different values for adjustment speeds and attract different types of migrants. A city with a open economy system will have a high value for capital adjustment speeds which attracts high skilled workers while a city with a closed economy system will have a low value attracting low skilled workers. By disaggregating, we will have a better way to explain those local average effects.
2.3 The dynamic systems

2.3.1 Model (1): sticky wage and fixed capital stock in the short run and local stability of interior region

We consider a small open economy or region which produces one good requiring labor \((E)\) and capital \((K)\). Labor supply comes from two different sources within the given region \(i\): a fixed native labor force \((F_i)\) and a time-variant stock of net-migrants \((M_i(t))\). Capital stock \(K_i\) only accumulates from foreign direct investment (FDI) inflows coming from outside of this given region. Since we are mainly interested in the role of FDI, for the time being domestic investment is neglected. In our study, a representative firm in \(i\) has a well behaved production function \(G(E_{i,t}, K_{i,t})\) which is strictly concave in inputs\(^4\). At time \(t\) the firm in \(i\) decides the amount of inputs based on the profit maximization \(^5\). In the short run it faces a sticky real wage \((w_i)\) and fixed capital \((K_i)\) inputs, so employment demand \(E_i\) is determined as a function of the given \(w_i, K_i\) and so is also a constant\(^6\).

Here we analyse the case in which the migrants move between a developed region \(i\) (e.g. the urban area) and a developing or undeveloped area \(j\) (e.g. the agricultural area) and labour movements in either direction (immigration and return migration) occur when the net gain of relocation is positive. The net gain of immigration from \(j\) to \(i\) is determined by the expected income in \(i\) (employment rate\(\ast\)the real wage rate), the real wage rate in \(j\) \((w_j)\) and the moving cost \((T_{i,j})\). Return migration is treated asymmetrically and is determined by just the wage differential allowing for the incidence of moving costs, \(w_i + T_{ij} - w_j\). For reasons given below, we assume that only the employed in \(i\) can consider return migration and any return migrant is sure to find employment in \(j\). If expected movement in either direction cannot cover the moving cost, then there is immobility: no one moves. This gives us a piecewise but continuous differential equation for migration between \(i\) and \(j\):

\[
M_i(t) = \begin{cases} 
\delta_{m,i}[\min(1, \frac{E(K_i, w_i)}{(M_i(t)+F_i(t))})w_i - T_{i,j} - w_j], & \text{if positive (a)} \\
\delta_{m,i}[w_i - w_j + T_{i,j}], & \text{if negative (b)} \\
0, & \text{otherwise (c)} 
\end{cases}
\]

where the coefficients \(\delta_{m,i}, \delta'_{m,i}\) reflect the adjustment speeds of immigration and emigration respectively. Considering the units, \(\delta_{m,i}, \delta'_{m,i}\) refer to the number of migrants per year per dollar. If the wage gap increases by one dollar of the variable, then there will be \(\delta_{m,i}\) or \(\delta'_{m,i}\) more immigrants.

Part (a) of the equation defines the immigration process in which the gap between the expected real wage \((\min(1, \frac{E(K_i, w_i)}{(M_i(t)+F_i(t))})w_i)\) in \(i\) and the forsaken real wage \((w_j(t))\) in the origin location \(j\) is high enough to cover the

\(^4\)The strict concavity assumption leads to the following properties: marginal product of labor \(G_E > 0\); marginal product of capital \(G_K > 0\); \(G_{EE} < 0\); \(G_{KK} < 0\); \(G_{KK} \ast G_{EE} - G_{EK} \ast G_{KE} > 0\).

\(^5\)\(\pi(P_{t,t}, w_{i,t}) = \max_{E_{i,t} \geq 0} P_{t,t} G(E_{i,t}, K_{i,t}) - W_{i,t} \ast E_{i,t} = \max_{E_{i,t} \geq 0} G(E_{i,t}, K_{i,t}) - W_{i,t} \ast E_{i,t}\), Where \(P_{t,t}\) is the nominal price of the output and the real wage rate \(w_{i,t}\) equals to \(\frac{W_{i,t}}{P_{t,t}}\).

\(^6\)We get a fixed employment demand \(E^*_i = E(K_i, w_i)\). Also \(E^*\) exists and is unique \(Iff \lim_{E \to -\infty} G_E(E_i, K_i) < w_i\) and \(G_E|_{E=0} > w_i\). As the firm will continue to increase the labor demand until \((G_E)\) reaches real urban wage rate at equilibrium.
moving cost \( (T_{i,j}) \). For region \( i \) the employment rate (chance of a job) in \( i \) is \( \min[1, \frac{E_i}{(F_i + M_i)}] \) so that at time \( t \) if there is excess demand for labour, then there is certainty of employment in \( i \). The "rest of the world" is modelled as region \( j \). We think of \( j \) as an undeveloped largely agricultural economy with no clearly functioning labour market but in which it is always possible to gain a subsistence income\(^7\) \( w_j \). So in \( j \) the expected income available is \( w_j \) for sure. For example, a migrant working in the farmland in region \( j \) is always employed. This idea is consistent with Harris-Todaro’s (1969, 1970) rural-urban migration in a two sector setting for developing countries. For the link between region \( i \) and \( j \), individuals face a moving cost \( T_{ij} \). The moving cost \( T_{ij} \) can be either the cost of travelling the physical distance or the entry cost in \( i \) or the exit cost in the "rest of the world" \( j \). For example, the costs of acquiring a Hukou is part of the entry cost for migrants in China.

Part (b) of the equation represents the return migration\(^8\) case in which the wage difference \( w_i - w_j \) exceeds the moving cost for those employed individuals in region \( i \). The emigration from \( i \) to \( j \) continues so long as \( w_j - w_i - T_{ij} > 0 \). For various reasons, we model the immigration case and the return migration case in an asymmetric way and select the employed workers to be the return migration candidates \( (w_i \text{ instead of } \min(1, \frac{E(K_i, w_i)}{(M_i(t) + F_i(t)))} w_i) \).

Firstly, the unemployed workers\(^9\) are claimed to be more credit constrained and have no or fewer resources to cover the moving cost (Docquier & Rapoport, 2010). Secondly, the reasons why the return migrants have a sure chance of employment in \( j \) is because they are higher skilled, they are capital rich and can establish businesses, they can access efficient social networks to secure employment in \( j \). The empirical importance of networks is confirmed by Dustmann and Kirchkamp (2002) and Kugler and Rapoport, (2006). According to their findings, return migrants face 100% employment probability in \( j \). Another fundamental reason for asymmetry in the immigration and emigration processes is the different impact of moving costs. A move in either direction is only undertaken if the gain from the move covers the costs \( T_{ij} \). But this means that \( T_{ij} \) enters with opposite sign in the immigration and return migration processes, thus ensuring that the moving cost wedge creates an interval of wage differentials within which there is immobility. Part (c) of the equation displays this immobility case in which either the moving cost is too high or the employment rate is too low for migrants so that the neither the net gains of immigration or return migration are positive.

Given that the only time dependent variable is \( M_i(t) \) and \( w_i \) is sticky over time, the conditions for the existence of a stationary population level (immobility) vary with the values of the fixed wage rates, employment demand and

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\(^7\)This highlights the importance of asymmetric immigration and emigration possibilities arising from heterogeneous labour market systems in origin and destination.

\(^8\)The determinants of return migration are coherent with those (high real wage rate in \( j \) and the cost of moving) defined by Stark (1996).

\(^9\)If instead jobs in \( i \) were renegotiated each instant then the dynamic determinant of emigration would be \( \min(1, \frac{E(K_i, w_i)}{(M_i(t) + F_i(t)))} w_i - w_j + T_{ij}) \). The qualitative properties we find below would not change if we used this formulation.
moving cost.

When \( w_i, K_i \) are time invariant (Figure 7)

**Proposition 1**

(i) if \( w_i < w_j - T_{i,j} \), there is no stationary population distribution but continuous return migration;
(ii) if \( w_j - T_{i,j} \leq w_i \leq w_j + T_{i,j} \), there is a stationary population distribution and continuous immobility and there are an infinite number of stationary states with immobility; (iii) if \( w_i > w_j + T_{i,j} \), there is continuous immigration (the asymptotic level of immigration depends on the limit of \( \frac{E_i}{M_i + F_i} w_i \) as \( w_i \to \infty \) (see Fig 7).

If we observed a sample of net migration from this process we should see weakly monotone net migration with no sign reversals. The requirements of a constant real wage and constant capital stock limit the applicability of this special model but it could fit a relatively static centrally planned economy, perhaps such as North Korea or even some relatively under-developed economies of Latin America or sub Saharan Africa.

### 2.3.2 Model (2): fixed capital stock but flexible real wage rate in medium run

The urban region \( i \) has a well structured labour market, so one would expect the real wage rate \( w_i \) to adjust according to labour market conditions at least in the medium run. Here we analyse the case in which the real wage rate varies according to the excess demand for labour, migration flows are determined by expected income differences as in model (1) but the capital stock is still fixed. This gives a two dimensional system for the dynamic interaction between net migration and the real wage rate. The employers demand for labor \( (E_i(t) = E(K_i, w_i(t))) \) in each time period depends on the real wage rate \( (w_i) \) and on the fixed amount of capital \( (K_i) \) and so is time varying. The supply of labour at \( t \) is \( M_i(t) + F_i \) and so the wage rate adjusts proportionally to \( E_i(t) - (M_i(t) + F_i) \).

If the labour market clears, the real wage remains constant.
The two dimensional system is

\[
\begin{align*}
\dot{w}_i(t) &= \delta_{w,i}[E(K_i, w_i(t)) - (M_i(t) + F_i)] \\
M_i(t) &= \begin{cases} \\
d_{m,i}[\min(1, \frac{E(K_i, w_i(t))}{(M_i(t) + F_i)}) w_i - T_{i,j} - w_j], & \text{if positive} \\
\delta_{m,i}[w_i - w_j + T_{i,j}], & \text{if negative} \\
0, & \text{otherwise}
\end{cases}
\end{align*}
\]

where $\delta_w > 0$ is the adjustment speed for the real wage rate in $i$.

In the real wage/migration space we define the no-immigration locus by values of $w_i, M_i$ satisfying $\min(1, \frac{E(K_i, w_i)}{(M_i + F_i)}) w_i = T_{i,j} + w_j$. And we define the constant real wage rate locus by values of $w_i, M_i$ satisfying $E(K_i, w_i) = M_i + F_i$. Depending on the elasticity of the labor demand, the no-immigration line (line 2) is either upward or downward sloping (figure 2 and 3). The slope of this no-immigration line (line 2) will not affect the relative location of the three regimes. Region I (immigration) which is located to the right of the high real wage threshold ($w_i > w_j + T_{i,j}$) and to the downside of the no-immigration line has positive immigration flow ($M_i(t) > 0$) with the net gain of relocation from $j$ to $i$ being positive; the region R (return migration) which is located to the left of the low real wage threshold ($w_i < w_j - T_{i,j}$) has positive return migration flow ($M_i(t) < 0$). Apart from those two regions, the remaining areas refer to the immobility region. The immobility region includes the area above the no-immigration line (line 2) and to the left of the high real wage threshold ($w_i > w_j + T_{i,j}$) and the area between the high and low real wage thresholds ($w_j - T_{i,j} < w_i < w_j + T_{i,j}$).

**Proposition 2** The no-immigration line is upward (downward) sloping if labor demand is inelastic (elastic).\(^{10}\)

**Proposition 3** In both cases (elastic and inelastic), there is an infinite number of stationary states for migration and the real wage rate lying on a line of the constant wage line (S1-S2) satisfying $E(K_i, w_i) = (M_i + F_i)$ and $w_j - T_{i,j} < w_i < w_j + T_{i,j}$ at the fixed $K_i$.

The directions of change vary at different points $w_i$ and $M_i$ (figure 9 and figure 8). When there is excess labor supply, the real wage will be decreasing (the horizontal arrows above the employment line 1 are pointing inward). When there is excess labor demand, the real wage will be increasing (the horizontal arrows below the employment line 1 are pointing outward). At a high real wage in $i$ the positive net gain of immigration drives a large number of immigrants to move from $j$ to $i$ (the vertical arrows below the no-immigration line 2 are pointing upward).

\(^{10}\) The no-immigration line ($M_i(t) = \frac{E(K_i, w_i(t)) + w_i(t)}{w_j + T_{i,j}} - F_i$) is downward sloping when labour demand is elastic.

\[
\frac{dM_i(t)}{dw_i(t)} \left(\frac{dE}{w_i} + F_i\right) = \frac{E(\frac{dE}{w_i} + \frac{w_i}{w_j + T_{i,j}})}{w_j + T_{i,j}} = \frac{E(\eta + 1)}{w_j + T_{i,j}}, \text{where } \eta \text{ is the labour demand elasticity. If } \eta < -1 (|\eta| > 1), \text{ the labour demand will be elastic. Then } \frac{dM_i(t)}{dw_i(t)} = \frac{E(\eta + 1)}{w_j + T_{i,j}} < 0, \text{ the no-immigration line will be downward sloping. Otherwise, the no-immigration line will be upward sloping.}
Conversely with a relatively low real wage in $i$ but the high real wage in $j$ and a low moving cost, employed workers in $i$ will be motivated to return to $j$. So that there will be a positive return migration flow (the vertical arrows to the left of $(w_i = w_j - T_{ij})$ are pointing downward). However the gains to moving do not cover the migration cost if $|w_i - w_j| < 2T_{ij}$ or above the immigration line where the stock of migrants is so high in $i$ that the chance of getting a job there is too low. For all such $w_i, M_i$ combinations there is no incentive to move. Piecing this information together gives cyclical paths, crossing between the regions of immigration, return migration and immobility.

Migration flows have different determinants in the different regions of the $w_i, M_i$ plane. However any dynamic path is continuous where it crosses from one region into another. We also have an infinite number of stationary points so that stability analysis of the equilibria is non-standard. We say that any stationary point $x$ is locally stable if there is an open set $S(x)$ containing $x$ and such that starting from any point $y$ within $S(x)$ that is not itself a stationary point, the path through $y$ converges to some stationary point. There is a small enough neighbourhood around any interior stationary point (contained entirely within the immobility region) such that starting anywhere in this neighbourhood, the solution path will converge to another stationary point and will never cross the lower threshold.\(^\text{11}\)

The most interesting points are at the ends of $S_1 S_2$. For these we can adapt the usual arguments of local stability analysis by computing eigenvalues of the system subject to different dynamics in small areas around $S_1, S_2$. We

\(^{11}\)In more technical terms in a neighbourhood of an interior stationary point contained within the immobility region, local dynamics are determined by the immobility Jacobian which is always locally stable.

\[
\text{immobility} \rightarrow \begin{bmatrix} 1/G_{EE} & -1 \\ 0 & 0 \end{bmatrix}
\]

Determinant $= 1 > 0$; trace $= 1/G_{EE} < 0$. 

Figure 8: Inelastic labour demand
can partition a neighborhood $N_1$ of the stationary point $S_1$ and a neighborhood $N_2$ of the stationary point $S_2$ into three sub-neighbourhoods, each contained within one of the three regions, for example (Fig 10) $SM_4 \cup SM_3 = N_1$ and $SM_1 \cup SM_2 = N_2$. Paths which start in $SM_1$ follow immigration dynamics, in $SM_4$ return migration dynamics and in $SM_2, SM_3$ immobility but with a varying wage.

We can then determine local stability of the stationary point by computing the eigenvalues of each part of the dynamic process at the relevant stationary point. For paths starting close to $S_2$ and in the immigration regime $SM_1$, the Jacobian matrix becomes

\begin{align}
\textit{immigration} \rightarrow \begin{bmatrix}
\frac{dE}{dw_i} & -1 \\
\frac{1}{M+F} & 1 \\
\end{bmatrix} = \begin{bmatrix}
\frac{1}{G_{EE}} & -1 \\
\frac{1}{(M+F)^2} & 1 \\
\end{bmatrix} \quad \text{if } E(K_i, w_i(t)) < 1 \\
\textit{immigration} \rightarrow \begin{bmatrix}
\frac{dE}{dw_i} & -1 \\
\frac{1}{M+F} & 1 \\
\end{bmatrix} = \begin{bmatrix}
\frac{1}{G_{EE}} & -1 \\
\frac{1}{(M+F)^2} & 1 \\
\end{bmatrix} \quad \text{if } E(K_i, w_i(t)) > 1
\end{align}

where $\eta$ is the wage elasticity of employment demand. Evaluating (matrix (3)) at $S_2$

\begin{align}
\begin{bmatrix}
\frac{1}{G_{EE}} & -1 \\
\frac{1}{(M+F)^2} & 1 \\
\end{bmatrix} = \begin{bmatrix}
\frac{1}{G_{EE}} & -1 \\
\frac{1}{(\eta + 1)(M+F)^2} & 1 \\
\end{bmatrix}
\end{align}

so the two eigenvalues have negative real parts\textsuperscript{12}. The Jacobian evaluated at $E(K_i, w_i(t))/(M_i(t) + F_i) > 1$ also has two roots with negative real parts\textsuperscript{13}.

\textsuperscript{12}The determinant is $\eta + 1 - w_i/(G_{EE}E) = \eta + 1 - (\eta/w_i) * w_i = 1 > 0$
and the trace is $1/G_{EE} - 1/E < 0$.
\textsuperscript{13}Determinant is $1 > 0$; trace is $dE/dw_i < 0$. 

Around $S_1$ in the subneighbourhood $SM_4$, the Jacobian for return migration dynamics is
returnmigration → \[
\begin{bmatrix}
\frac{dE}{dw_i} & -1 \\
1 & 0
\end{bmatrix}
\]
which is locally stable (the two eigenvalues have negative real parts\(^{14}\)). In the subneighbourhood \(SM_3\) the immobility dynamics hold and we know those are locally stable\(^{15}\).

**Proposition 4** Stationary points interior to the immobility region are always locally stable. Given that the return migration system (\(SM_3\)) and immobility (\(SM_4\)) cases around \(S_1\) are locally stable, so the the point \(S_1\) is locally stable. Similarly the immigration system (\(SM_1\)) and immobility (\(SM_2\)) cases are locally stable so the point \(S_2\) is locally stable.

When the moving cost is zero \((T_{ij} = 0)\) the local stability analysis is easier to visualize and formalise (Figure 12 and figure 11). Due to the zero moving cost, the immobility region \((w_j - T_{ij} \leq w_i \leq w_j + T_{ij})\) converges to a single line \((w_j = w_i)\). But as a result of the bounded employment probability and the moving cost, the dynamic process of migration still differs between the immigration region, return migration and immobility regions. There is a unique stationary point \((w_i^*, M_i^*)\) at the intersection of these three regions. We apply the same half neighborhood methodology to study the local stability for \((w_i^*, M_i^*)\). Except for the change in the functional form of the dynamic

\(^{14}\) The determinant is \(1 > 0\) and the trace is \(dE/dw_i < 0\).

\(^{15}\) Determinant = \(1 > 0\); trace = \(1/G_{EE} < 0\).
system and the change in the partition of the neighborhood $N$ of the stationary point ($SM_1 \cup SM_2 \cup SM_4 = N$), the relevant Jacobian matrices stay the same and the local stability is unchanged.

Time series data from this process will have a richer structure than in the case with a fixed real wage (Figure 13). The correction between $M$ and $w$ could be either sign. For instance, $M$ could be positively and then negatively correlated with the wage rate in return migration region. People do not stop emigrating until the wage rate rises to the low threshold ($w_i \geq w_1; w_1 = w_j - T_{ij}$). Afterwards, people do not immigrate until the wage rate rise to the high threshold ($w_i \geq w_2; w_2 = w_j + T_{ij}$) so that the wage rate is high enough to compensate the moving cost $T_{ij}$ and forgoing $w_j$. Hence, the migration flow becomes positive. The flow continues to be positive so long as the employment rate is not too low, even though the wage rate starts to decrease at some point. In the immigration region, the correlation between migration and the wage rate could be either sign till it converges to a stationary state. In Fig 13, we show the dynamic path converges to a stationary equilibrium after a short run. At a stationary state, $dw_i/dt = 0$ and $dM_i/dt = 0$ conditional on fixed $M_i$.

\[
M_i(t) = \begin{cases} 
\delta_{m,i}[\min(1, \frac{E(K_i, w_i(t))}{(M_i(t) + F_i)})]w_i(t) - w_j, & \text{if positive} \\
\delta_{m,i}[w_i(t) - w_j], & \text{if negative} \\
0, & \text{otherwise}
\end{cases}
\]

\[
w_i(t) = \delta_{w,i}[E(K_i, w_i(t)) - (M_i(t) + F_i)]
\]
Figure 12: Elastic labour demand (T=0)

Figure 13: Time series for M and W
2.3.3 General framework (model (3)): flexible capital and real wage rate

Finally, we relax the assumption of a fixed capital input. FDI flows respond to the differential between the return in $i$ (the marginal product of capital) and the world real interest rate $r$. Each of the three variables adjusts to its own partial equilibrium level with $\delta_{x,i} > 0$ ($x = w, M, K$) the adjustment speed for location $i$. The dynamic system becomes,

$$
\begin{align*}
M_i(t) &= \begin{cases} 
\delta_{m,i}[\min(1, \frac{E(K_i(t), w_i(t))}{(M_i(t) + F_i)})]w_i - T_{i,j} - w_j, & \text{if positive (IM)} \\
\delta_{m,i}'[w_i(t) - w_j + T_{i,j}], & \text{if negative (RM)} \\
0, & \text{otherwise}
\end{cases} \\
w_i(t) &= \delta_{w,i} [E(K_i(t), w_i(t)) - (M_i(t) + F_i)] \\
K_i(t) &= \delta_{k,i} (MPK_i(t) - r)
\end{align*}
$$

(4)

where $MPK_i(t)$ is the marginal product of capital of region $i$ and $r$ is the world interest rate at time; $K_i(t)$ is the capital (FDI) inflow.

Since capital is subject to diminishing returns, capital flows continue until the marginal product of capital $(MPK_i)$ is equal to the world interest $(r)$. The marginal product of capital is determined by the technology and labor. In particular the impact of one input on the demand for the other depends on the substitute/complementarity relation between them. If capital and labor are complements, an increase in the capital stock leads to an increase in labor demand. This increase in labor demand shifts the demand curve upward at any real wage, so that more jobs will be created. In turn this raises the probability of employment and attracts more immigrants from $j$ to $i$. But at the same time, the real wage rate is determined by the labor supply. If the number of immigrants is far higher than the number of job vacancies, there will be excess labor supply which leads to the real wage rate falling. If labor and capital are substitutes, an increase in the capital stock will lead to a decrease in labor demand shifting the demand curve downward, so that more jobs will be destroyed and the employment rate will fall. Then the employed workers in $i$ will be pushed to return to $j$. If the number of return migrants is very high and over-reacts, then the real wage rate may rise because of inadequate labor supply.

Similar to the $w/M$ space, in the $w, K$ and $M$ space, the definitions for the no-immigration locus and constant real wage rate locus stay the same as in model (2). In addition, here we define the capital locus by values of $w, K$ and $M$ and $r$ satisfying $MPK_i = r$. Also the regions for immigration, return migration and immobility are very similar to those in the two dimensional system, except that each of the three regimes are partitioned into two parts by the capital locus. Within the two elements of each partition there are different signs for capital inflows. The
parts located to the right part of the capital locus (with the high real wage rate) have positive capital inflows. While the parts located to the left part of the capital locus (with the low real wage rate) have negative capital inflows.

The dynamic paths in the 3 dimensional space for labor market and migration process are consistent with those in the 2 dimensional space. For example, the real wage is increasing (decreasing) when there is excess labor demand (supply) and there is a positive (negative) immigration flow when the net gain is positive (negative). In addition, the marginal capital product of capital is a decreasing function of the real wage rate. Capital inflows (outflows) to (from) region \( i \) when \( w_i \) is relatively low (high). Immigration takes off at a high wage rate and a low initial stock of migrants. Especially, in a region with a high employment probability and high real wage rate, the immigration flow will not stop until job creation (driven by capital) and the associated rise in the chance of employment can no longer compensate the relative real wage rate loss (i.e. the net gain of relocation becomes zero). However, a backward region with growing but low initial values for \( w_i \) and \( K_i \) will not attract immigration but will push people to leave from \( i \) to \( j \), despite its growing state. In addition, the labor market in a region with growing wage and capital stock but a high moving cost or a low employment probability will be in an equilibrium state of immobility.

**Proposition 5** The no-immigration surface is downward (upward) sloping if labor demand is elastic (inelastic). Stationary states exist when the capital surface \((MPK_i(t) = r)\) intersects with the employment surface \((E(K_i(t), w_i(t)) = (M_i(t) + F_i(t)))\) within the immobility regions \((-T_{i,j} + w_j \leq w_i \leq T_{i,j} + w_j)\) \((M_i(t) = 0)\) (see two examples: figure 14 and figure 15).

Example a : stationary states do not exist when the capital surface (yellow) intersects with the employment surface (green) outside of the immobility region (figure 14). In figure 14, the two red planes refer to two thresholds for immigration \((w_i > T_{i,j} + w_j)\) and return migration \((w_i < -T_{i,j} + w_j)\). Any space within the red planes \((-T_{i,j} + w_j \leq w_i \leq T_{i,j} + w_j)\) is the immobility caused by moving cost. The yellow surface refers to the capital surface, the green surface refers to employment surface and the blue surface refers to immigration surface. Any space to the right of the immigration threshold \((w_i > T_{i,j} + w_j)\) but above the blue surface \((\min(1, \frac{E(K_i(t), w_i(t))}{(M_i(t)+F_i)})w_i - T_{i,j} - w_j = 0)\) is the immobility region caused by the low employment probability. When the capital surface (yellow) intersects with the employment surface (green) outside of the two red planes, the stationary states do not exist because the intersection line is within the immigration region\(^{17}\).

Example b : Stationary states exist when the capital surface (yellow) intersects with the employment surface (green) within the immobility region. In figure 15, the area between the two red planes \((-T_{i,j} + w_j \leq w_i \leq T_{i,j} + w_j)\)

\(^{17}\)Let \( dM_i(t)/dt = \frac{E(K_i(t), w_i(t))}{(M_i(t)+F_i)} \) \( w_i - T_{i,j} - w_j = 0 \), we get \( M_i(t) = \frac{E(K_i(t), w_i(t))}{K_{i,j}+w_j} w_i - F_i \). Let \( dw_i(t)/dt = E(K_i(t), w_i(t)) - M_i(t) - F_i = 0 \), we get \( M_i(t) = E(K_i(t), w_i(t)) - F_i \). Above the right threshold, \( w_i > T_{i,j} + w_j \), we have \( \frac{E(K_i(t), w_i(t))}{K_{i,j}+w_j} w_i - F_i > E(K_i(t), w_i(t)) - F_i \). The no-immigration surface is always above the employment surface.
is the immobility region. So long as the intersection line between the capital and employment surfaces (yellow and green) is within the immobility region, stationary states exist.

**Proposition 6** If stationary states exist, there will be an infinite number lying on a line defined by $-T_{i,j} + w_j \leq w_i \leq T_{i,j} + w_j$; $E(K_i, w_i(t)) = (M_i(t) + F_i(t))$ and $MPK_i = r$ in the three dimensional space.

As in the propositions presented above, if there are stationary points, there are an infinite number of stationary states lying on a line ($S_1 - S_2$) in the employment surface (Fig 16). The stationary point $S_1$ is the intersection point of the return migration set and immobility set and the stationary point $S_2$ is the intersection point of the immigration set and the immobility set. However if the intersection of the employment and capital surface have no points in common with the immobility region, then there are no stationary points.

The real wage is increasing (decreasing) when there is excess labor demand (supply) above the real wage surface (surface 1) and there is a positive immigration flow when the expected net gain is positive (above the real wage surface but below the no immigration surface 2) but emigration when the net gain is negative (at wages below the lower threshold). In addition, the marginal product of capital is a decreasing function of both the real wage rate and the capital stock. At a low wage or capital stock, productivity of capital is high and there inward flows of FDI. Conversely with high wages and capital stock there are FDI outflows. The immobility region is now defined by the area between the wage thresholds and above the no immigration surface 2.
Figure 15: Example b

Figure 16: Stationary points S1 S2
Figure 17: 3D dynamic path 1

The solution paths are still continuous in this model but again can display kinks where the dynamic regime changes. In a low wage/high migration stock region there is return migration, a brief period of immobility and immigration and then immobility. There may be convergence to a stationary state or the cycle may repeat (fig 17 and fig 18). From a high wage and a moderate migration stock there is a brief region of immigration followed by immobility and then return migration, eventually converging to a stationary state. With a high wage, high migration stock there is labour immobility and a falling wage (capital can either rise or fall) followed by return migration and convergence to a stationary state. The configuration of migration regimes is similar to the fixed capital case, but the flexibility of capital adds adjustment possibilities between both migration and capital and between the real wage rate and capital. In particular within the immobility region \( dM/dt = 0 \) but there are dynamics in \( K, w_i \) around the \( S_1S_2 \) line. These dynamics in just \( K, w_i \) are always locally stable for a fixed migration stock and are typically cyclical in \( w, K \). Typical evolution along a convergent path starting from a low net immigrant stock is a period of immobility with rising real wages and capital inflow into the destination followed by immigration with at first still rising wages and capital. But once the migrant stock rises sufficiently wages and capital stock start falling (capital outflow) and we enter immobility. As soon as this happens there are only dynamic movements in \( K, w \) which may be convergent to a stationary state or may lead to \( w, K \) values pushing the system below the lower wage threshold. If this happens a period of return migration follows. In Fig 17, there as an extended example of the same type of path with several tree dimensional cycles before entering the immobility region and following convergent cycles in \( K, w \) with \( M \) constant. In this final phase there are convergent cycles in \( K, w \). Fig 18 shows an example of a path starting with high values of all the variables which is initially in immobility above the no-immigration surface. It proceeds to a phase of return migration, then re-entering immobility followed successively by immigration and immobility.
Paths are subject to different dynamics in different parts of the phase space. However any dynamic path is continuous where it crosses from one region into another. We also have an infinite number of stationary points so that stability analysis of the equilibria is non-standard, we follow the approach used in the fixed capital case. Again any stationary point in the interior of $S_1S_2$ is locally stable in our sense, the formal argument of the fixed capital case applies here$^{18}$.

The most interesting points are $S_1, S_2$. We use the half-neighbourhood methodology to study the local stability of $S_1$ and $S_2$. Exactly as in the fixed capital case, we can partition a neighborhood $N_1$ of the stationary point $S_1$ into parts $SM_4$ which follow return migration dynamics and a part $SM_3$ which follows immobility dynamics. Linearising the return migration dynamics around $S_1$, any point in $SM_4$ will follow a linear system with Jacobian

$\begin{bmatrix}
G_{KK} - \frac{(G_{KE})^2}{G_{EE}} & \frac{G_{KE}}{G_{EE}} & 0 \\
\frac{-G_{KE}}{G_{EE}} & \frac{1}{G_{EE}} & -1 \\
\frac{G_{KE}}{G_{EE}} & \frac{1}{G_{EE}} & 0
\end{bmatrix}$

Starting from a point near $S_1$ with return migration, $S_1$ is locally stable$^{19}$. On the other hand any point in $SM_3$

$\begin{bmatrix}
G_{KK} - \frac{(G_{KE})^2}{G_{EE}} & \frac{G_{KE}}{G_{EE}} & 0 \\
\frac{-G_{KE}}{G_{EE}} & \frac{1}{G_{EE}} & -1 \\
\frac{G_{KE}}{G_{EE}} & \frac{1}{G_{EE}} & 0
\end{bmatrix}$

The final row of zeros means that one root is zero and the remainig two coincide with those of the principal minor $M_{33}$ which has a negative trace, zero determinant and positive sum of principal minors.

$^{18}$The Jacobian evaluated at any stationary point interior to $S_1-S_2$ is

$\begin{bmatrix}
G_{KK} - \frac{(G_{KE})^2}{G_{EE}} & \frac{G_{KE}}{G_{EE}} & 0 \\
\frac{-G_{KE}}{G_{EE}} & \frac{1}{G_{EE}} & -1 \\
\frac{G_{KE}}{G_{EE}} & \frac{1}{G_{EE}} & 0
\end{bmatrix}$

The determinant is $(G_{KK}G_{EE}-G_{KE}^2) < 0$ and the trace is $(1+G_{KK}G_{EE}-G_{KE}^2) < 0$. The principal minors are $M_{11} = 1, M_{22} = 0, M_{33} = (G_{KK} - \frac{(G_{KE})^2}{G_{EE}}) + (\frac{G_{KE}}{G_{EE}})^2 = \frac{G_{KK}}{G_{EE}}$ so the sum of principal minors is positive.

$^{19}$
follows immobility dynamics and the relevant Jacobian is

\[
\begin{bmatrix}
G_{KK} - \frac{(G_{KE})^2}{G_{EE}} & \frac{G_{KE}}{G_{EE}} & 0 \\
-\frac{G_{KE}}{G_{EE}} & \frac{1}{G_{EE}} & -1 \\
0 & 0 & 0
\end{bmatrix}
\]

which is also locally stable. So paths starting in SM3 converge to S1. Hence locally any path starting near S1 converges to S1.

Similarly a neighborhood N2 of the stationary point S2 can be partitioned into two sub-neighbourhoods, SM2 contained entirely within the immobility region and SM1 contained entirely within the immigration region, SM1 ∪ SM2 = N2. Paths which start in SM1 follow immigration dynamics, in SM2 immobility but with varying wage and capital stock. We can then determine local stability of the stationary point by computing the eigenvalues of each part of the dynamic process at points close to S2. A path starting in SM1 has a Jacobian matrix

\[
\begin{bmatrix}
G_{KK} - \frac{(G_{KE})^2}{G_{EE}} & \frac{G_{KE}}{G_{EE}} & 0 \\
-\frac{G_{KE}}{G_{EE}} & \frac{1}{G_{EE}} & -1 \\
\frac{1}{(M+F)w_i} & \frac{1}{M+F} & \frac{1}{w_i}
\end{bmatrix}
\]

when evaluated at S2. This has a negative trace and a determinant given by

\[
\frac{(G_{KK}G_{EE} - G_{KE}^2)((1 + \eta)\eta - 1)}{\eta G_{EE}}
\]

The sum of the principal minors is

\[
\frac{G_{KK}}{G_{EE}} = \frac{1 - \eta(1 + \eta) + G_{KK}G_{EE} - (G_{KE})^2}{\eta}
\]

If \((1 + \eta)\eta - 1 < 0\) the sum of principal minors is positive and the determinant is negative. Thus any path starting in SM1 will be locally stable if \((1 + \eta)\eta - 1 < 0\). This holds if \(\eta > -0.5(1 + \sqrt{3})\). Similarly starting from any point in SM2 with immobility dynamics locally the movement follows the same Jacobian as in SM3 and is locally stable.

**Proposition 7** (1) S1 is locally stable; (2) If \((1 + \eta)\eta - 1 < 0\), S2 is locally stable

---

20 The determinant is 0 and the trace is \(\frac{(1+G_{KK}G_{EE}-G_{KE}^2)}{G_{EE}} < 0\). The sum of principal minors, \(G_{KK}/G_{EE}\), is positive.

21 The principal minors are

\[
M_{33} = (G_{KK}G_{EE} - (G_{KE})^2) + (G_{KE})^2 = G_{KK}G_{EE} = G_{KK}/G_{EE},
\]

\[
M_{11} = -\frac{w_i}{G_{EE}E} + (1 + \eta),
\]

\[
M_{22} = -\frac{(G_{KK}G_{EE} - (G_{KE})^2)w_i}{E}
\]

---

34
If $T_{ij} = 0$, what will happen to the stationary state? In general, the function form of the system is changed because of the zero cost and with $T_{ij} = 0$ again the set of stationary points shrinks to at most one point. Assuming it exists, its local stability can be analyzed by the "half neighborhood" approach we used above. The stationary point $(w_i^*, M_i^*, K_i^*)$ is at the intersection of the immigration, return migration and immobility regions. If $(1 + \eta)\eta - 1 < 0$, the stationary point $(w_i^*, M_i^*, K_i^*)$ is locally stable.

With capital and real wage flexibility, the time series (figure 19) of $M$ and $w$ remain the similar patterns as in 2 dimensional spaces. Figure 19 portrays one generic pattern of the variables (from the upper middle immobility region into the return migration region, crossing into immigration region and back to the middle immobility region) in the three dimensional space. In this generic pattern, the dynamic path converges towards a stationary equilibrium conditional on the fixed net migration level. In the middle immobility region, the migration flow is zero and $K$ and $w$ show a cyclical pattern till the dynamic path converges to a stationary equilibrium. This can also be seen in figures 17 and 18. During this convergence, $w$ and $K$ could move in the same or opposite directions. In Fig 19, we show a positive correlation between $K$ and $w$ when $dM(t)/dt = 0$. Over a particular short horizon such as immigration periods (positive immigration flows), capital and the wage rate tend to show a positive correlation when the dynamic path is underneath the employment surface (surface 1 in Fig 16) but in periods of zero migration flows, the correlation between the two could be of either sign.

### 2.4 Empirical application for Guangdong

Since the open door policy in China and dramatic reforms later in 1978, Guangdong has been singled out as a province for regional development and has experienced the highest volume of cross province net migration (Chinese Population Census, 1990; 1995; 2000; 2005; 2010) within China and in the emerging world (UN, 2011). Especially, the Guangdong SEZs (special economic zones\(^{22}\)), ETDZs (Economic and Technology Development Zones) and COAs

\[ M_i(t) = \begin{cases} \delta_{m,i}[\min(1, \frac{E(K_i(t),w_i(t))}{M_i(t) + F_i})w_i(t) - w_j), & \text{if positive} \\ \delta_{m,i}[w_i(t) - w_j), & \text{if negative} \\ 0, & \text{otherwise} \end{cases} \]

\[ w_i(t) = \delta_{w,i}[E(K_i(t), w_i(t)) - (M_i(t) + F_i)] \]

\[ K_i(t) = \delta_{k,i}(MPK_i(t) - r) \]

\(^{22}\)The SEZs were chosen as a result of convenient communication and transportation capabilities from and to overseas countries, especially Macao and Hongkong (Ateno, 1979).
(Coastal Open Areas) 

Figure 19: Time series for M, K and w

with favorable government industrial development incentives have attracted a large number of Chinese workers from outside of Guangdong. In addition, rapid industrialization has been facilitated by high FDI (foreign direct investment), especially that arising from the geographical and social proximity to Hong Kong. Subsequently, industrial and trade areas have flourished especially around the Pearl River, triggering further high levels of rural-urban immigration from outside of Guangdong. Our aims are: (1) to estimate the dynamic adjustment speeds of the capital flow, the real wage and the net migration flow; (2) to investigate whether different Guangdong cities share homogenous adjustment speeds; (3) to study the simultaneous interactions between FDI, the real wage and net migration flows; (4) to test whether the capital market in Guangdong was negatively influenced by the 1997 Asian financial crisis.

2.4.1 City Characteristics and Data Description

Our empirical estimation covers 21 years (1990-2010) for 16 city areas in Guangdong. For the first 10 years predictions of real wages, net migration and FDI based on a mixture of calibration and estimation will be compared
with the sample panel data and for the next 11 years the predicted data projects the dynamic path out of sample.

The sample data comes from the Guangdong Statistical Yearbooks for the period 1990-1999 for 16 city areas\(^{27}\) (Figure 20). From the descriptive statistics, the capital city 1 (Guangdong) and the SEZ city 2 (Shenzhen) are the most developed areas within Guangdong province. Especially as a result of the closeness of city 2 to Hongkong, city 2 attracts the highest FDI from Hongkong (Guangdong Statistical Yearbook 1990-1999). The cities within Guangdong cluster into economic groups. The cities centered in the middle are more advanced economy areas (eg, city 2: a financial center; city 10, 12,11 and 13: manufacturing sectors; city 1 and 3: ETDS, COAs). The northern cities are mountainous areas and have concentrations of heavy industry. The landscape of the southern cities is lowland but those cities mainly produce agricultural goods.

In our sample data (1990-1999), the net migration \((M)\) data only covers qianyi renkou not the floating population in China\(^{28}\). The qianyi renkou movement (migration) is a spatial movement between previous residence and current destination leading to a change in hukou status and is often identified with permanent migration. The employment \((E)\) data covers the three chief workplace organizations of each city - SOEs (state owned enterprises), UCEs (urban collective owned enterprises) and other units\(^{29}\). The average wage \((w_{i,t})\) of the urban collective owned enterprises of each city is taken as proxy variable\(^{30}\) for the city market wage. This \(w_{i,t}\) is then deflated by city level CPI to get

\(^{27}\) Guangdong is divided into a maximum of 21 city areas but one of these, Jieyang (city 20) was only established in 1992 taking over some parts of Shantou (city 4). Moreover some parts of Zhaozhou (city19) were formerly part of Shantou (city4) before 1992. We merge these 3 city areas into a single unit (city 22). In addition Yunfu (city21) formerly was part of Zhaoqing (city17) before 1994, so we merge Yunfu and Zhaoqing into a single unit (city 23). For the sake of the consistency of geographical units over our sample time period, these two merged units are dropped out of our sample. This leaves 16 city areas.

\(^{28}\) The floating population (liudong renkou) is a unique concept in China and measures the stock of past migrants who have retained their original hukou status. Liudong renkou is often identified with temporary migration. The qianyi renkou is a measure of flow and is defined as "individuals five years old or younger who have moved from one county to another within the past year and (a) whose hukou has changed to the place of residence at the previous year or (b) who had left their hukou location for more than one year" (Fan, 2008).

\(^{29}\) The other units includes units funded by entrepreneurs from Hongkong, Macau and Taiwan, foreign funded units, joint ventures, shareholding units and others (Statistical Yearbook, 1999).

\(^{30}\) This method coincides with Lee (1999), who uses the average wage of the UCEs as proxy variables for market wages to estimate
the real wage \( \frac{W_{i,t}}{CPI_{i,t}} = w_{i,t} \), where \( W_{i,t} \) is nominal value and \( w_{i,t} \) is real value). National average rural income (the rest of China in our theory) is deflated by the national CPI \( \frac{W_{j,t}}{CPI_{j,t}} = w_{j,t} \), where \( W_{j,t} \) is nominal value and \( w_{j,t} \) is real value). Due to the geographical closeness and ethnic ties between Guangdong and Hong Kong, the majority of FDI in Guangdong flows from Hong Kong. So the world interest rate \( r_{\text{world}} \) is replaced by the Hong Kong annual rate of return \( r_{\text{HK}} \) in HangSeng stock market. Capital stock \( K \) is derived from an initial capital stock, capital flows and a city common depreciation rate \( \theta \). The depreciation rate \( \theta \) for each city is obtained from the Guangdong Statistical Yearbooks for the period 1990-1999.

The scatter diagrams (Fig 21) obtained from our sample data over 1990-1999 explain the interactions between the real wage, the capital stock and the number of net migrants by city (no cross city effects). The immigrants have a strong propensity to migrate towards the Pearl River cities especially the capital (Guangzhou (1)) and the one SEZ (Shenzhen (2)), where the real wage and capital stock are high. The employed workers in city 12 have a relatively strong propensity to leave, where the real wage rate and capital stock are low. The other regions, in general, indicate positive correlations between \( w_{i,t} \), immigration \( M_{i} \) and \( K_{i} \). In particular regions (the Pearl River delta cities) with a high level of capital stock and a high real wage normally have a high net immigration. Conversely the labor intensive and low urban real wage cities (the northern mountainous and the southern agricultural regions) have a small number of net migrants and a low real wage rate and low level of capital stock. So roughly, the regions in Guangdong are heterogenous between clusters but are homogenous within the same cluster (e.g. city 1 and 2 form one cluster).

The time series plots of the sample data (Fig 25 - 33) show continuous growth of capital stock in most cities but at varying rates, however cities 9, 15 and 16 show initial growth but then a downturn in the later years of the sample. Although the sample is short the real wage indicates a cyclical pattern in at least half of the cities. In some cities (6, 9, 10 and 14) the net migration is positive but small and relatively flat indicating that they may be in a phase of immobility, but the other cities show positive growing immigration. Overall though, the sample period reflects a relatively short window of immigration. In terms of our theoretical framework, the data only covers part of the dynamics between the three variables.

For the production technology, we use a calibrated Cobb-Douglas \( G(K, E) = AK^\alpha E^\beta \) with \( A = 1, \alpha = 0.4705, \beta = 0.5295 \). Wu (2000) finds that the total factor productivity in Guangdong typically ranges from 0.9996 correlation between market wages and firm employment in SOEs in China.

\( \frac{(P_t - P_{t-1})}{P_{t-1}} \) is inflation rate for Hong Kong and is taken from world bank (http://www.worldbank.org/); \( rr \) is calculated by author and is taken from HangSengIndex (http://uk.finance.yahoo.com/q/hp?s=%5EHSI).

\( r_{\text{HK}} = HH_{K_t} = rr - \frac{(P_t - P_{t-1})}{P_{t-1}} \).
to 1.0605 from 1979 to 1997. Thus, $A = 1$ should be an appropriate value for Guangdong. Also his empirical estimation suggests that $\alpha = 0.4705$, $\beta = 0.5295$. No other studies on Guangdong cities are available, so we apply city-invariant values $\alpha$ and $\beta$ for all cities. In terms of the theoretical framework, a Cobb-Douglas always has an elastic labour demand and so we should expect to predict the corresponding cyclical patterns of the relevant theoretical phase diagrams and time series plots.

### 2.4.2 Empirical estimation of adjustment speeds

Given the values of two exogenous variables $rHK(t)$ and $w_j(t)$ over the period 1990-2010, it is now possible to calculate the dynamic path for capital stock ($K_i(t+1)$), real urban wage ($w_j(t+1)$) and the net migration stock ($M_i(t+1)$). The dynamic system starts from historically given actual values$^{33}$. The estimated dynamic system is an extension of the model (3) of equation (4),

$$K_{i,t} = (1 - \theta)K_{i,0} + FD_{i,0} = 0 + K_{i,0}; (\frac{K_i}{T_i})_{i,1} = (\frac{K_i}{T_i})_{i,0} + (\frac{K_i}{T_i})_{i,0}; T_i = 0.5w_{j,t}; M_{i,1} = M_{i,0} + M_{i,0}.M_{i,0} = 0.$$  

According to the statistic report from Guangdong Yearbook, $K_{i,t}$ is set equal to $2.8M_{i,t}$. The "open door" policy (Guangdong start to have SEZs) initiated in year 1991. This attracted a large amount of FDI from Hongkong and some south Asia countries (South Korea, Janpan etc), boosting labour demand in Guangdong. This further started to attract migrants. Thus, setting a zero value for $M_{i,0}$ for the initial year 1990 is a reasonable assumption.

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$^{33}$
\[ K_i(t+1) = (1 - \theta_i)K_i(t) + K_i(t) \]  
\[ w_i(t+1) = w_i(t) + w_i(t) \]  
\[ M_i(t+1) = M_i(t) + M_i(t) \]

\( i = 1..16; t = 0..20 \)

where \( K_i(t), w_i(t), M_i(t) \) are defined in the general model (model (3)); \( K_i(t+1), w_i(t+1) \) and \( M_i(t+1) \) are the capital stock, the real wage and the net migration stock at time \( t+1 \) for city \( i \); the index \( i \) stands for the 16 cities within Guangdong province.

The three adjustment speeds for each city are estimated by the method of moments (Sims, 2010; Ruge-Murcia, 2007), to match the means of the sample data to the predicted means of the model. Formally, let \( m^* \) be a \( k \times 1 \) vector of data moments. Let \( m(\Theta) \) be the vector of those same moments from the model evaluated at a given set of adjustment speeds such as \( \Theta = [\delta_1, \delta_2, \delta_3] \). Let \( W \) be a \( k \times k \) matrix, for simplicity we choose an identity matrix.

The GMM estimate of the model’s parameters (adjustment speeds) is

\[ \Theta^* = \arg\min (m(\Theta) - m^*)'W(m(\Theta) - m^*) \]  

The adjustment speeds for \( K, M \) and \( w \) effectively minimize the sum of squares of deviations between the model data and the first moments over 1990-1999. Given the estimated adjustment speeds, predictions of \( K, w \) and \( M \) come from the model for the sample period 1990-1999 and the projection period 2000-2010.

Figures 22 - 24 show that the adjustment speeds for capital and the net migration are consistent with the scatter diagrams (Fig 21), suggesting that city 1 and 2 can be clustered with rapid adjustments. The remaining cities are very different from these two cities and have low adjustment speeds. However, figure 14c shows that the adjustment speeds for the real wage rate in Guangdong are rather dispersed. Roughly, the advanced regions with high FDI (cities 1&2) still have high values for the adjustment rate of \( w_i \), manufacturing centers (city 3 and 13) have moderate values while the less developed regions share low values. An exception is cities 5 and 16 which have a low initial level of \( w_i \) but have high adjustment speeds for the real wage rate.

Note that in all cities the estimated adjustment speeds for each equation are positive as required by the theory. Empirically they are also all less than unity. The lowest estimated adjustment speed for the capital equation is in Heyuan(6) and Yangjiang (14) at a value of 0.003. The highest estimated adjustment speed for the capital equation is in city 1 and 2 at 0.95. The lowest adjustment speed for wage is in city 6 at 0.015. The lowest adjustment speed
Figure 22: Dynamic adjustment speeds for capital

Figure 23: Dynamic adjustment speeds for migration

Figure 24: Dynamic adjustment speeds for wage
For the net migration is in city 14 at 0.001.

For capital, the dynamic adjustment speed (0.95) is very high and is close to 1 in the capital intensive regions (city 1 and 2). A one unit increase in the capital return gap \( \alpha AK_{i,t}^{\alpha-1}(E_{i,t})^\beta - rHK_i \) leads to 9.5 billion yuan FDI inflows into Guangzhou and Shenzhen. In the moderately developed regions, the adjustment speed of FDI varies between 0.09 and 0.2. In manufacturing centers, the sensitivity of FDI inflow to the capital return gap \( \alpha AK_{i,t}^{\alpha-1}(E_{i,t})^\beta - rHK_i \) is moderate. The value of 0.2 is low but far bigger than that of the less developed regions, where FDI adjusts at a very slow speed (0.01 and 0.003). Particularly the values of the capital adjustment speed in Heyuan(6) and Yangjiang (14) are 0.003 so that capital only reacts marginally to a change in the capital return gap. So the adjustment speeds are heterogenous in Guangdong but are homogenous within the same cluster of cities. In the next 11 years the estimated model predicts that FDI will continuously flow from Hong Kong to Guangdong.

For the dynamic adjustment speeds of net migration, the capital Guangzhou(1) and the ETDZ (Shenzhen(2)) cities have relatively fast migration adjustment speeds (0.08 and 0.04 respectively) The SEZ cities (Zhuhai(3) and Foshan(13)) have moderate sensitivity to the net gain \( \frac{E_{i,t}}{M_{i,t}+F_{i,t}}(\frac{w}{p})_{i,t} - T_{i,j,t} - (wr)_t \). Region (Huizhou(8)) shows its potential attraction for rural migrants from outside of Guangdong. The remaining regions are not very sensitive to the wage gap and migration adjusts at slow speeds. An overall low adjustment speed for migration reflects institutional features such as barriers to internal labor movement in China. Recognizing the special Hukou system in China, permanent rural migration means a change of a resident’s household registration record from rural to urban areas during migration and this change is restricted by government.

The real urban wage in less developed regions shows more or less the same sensitivity (average adjustment speed 0.015) to the change in employment vacancies \( E_{i,t} - (M_{i,t} + F_{i,t}) \), except that the capital city (1:Guangzhou), the financial center (city 2) and the manufacturing cities (3, 13) are more sensitive to the change. Compared with the adjustment speeds for capital, the generally low value of the adjustment speed is a perfect reflection of sticky wage rates in China\(^{34}\). Real wages in the chief three employing organizations are not flexible and the sensitivity to the change in employment vacancies is generally low. The predictions show that this low sensitivity will continue in the next 11 years but the real wage will be growing slowly.

In general, we can see that the values of migration adjustment speeds are relatively high in cities for which FDI inflows and the real wage rate occur at a fast speed. This is in line with our hypothesis that FDI has a positive effect on migration and represents an incentive for migration. The increase in the accumulation of capital positively

\(^{34}\)This argument is consistent with Ning’s (2008) empirical findings. He uses the panel data of 31 province in China over 1993-2005 to analysis the level of flexibility of wage in China and he finds that the eastern costal areas’ (e.g. Guangdong) wage is sticky.
Figure 25: Calibrated capital 1

influences wage rates and migration. In our empirical analysis, the scatter diagrams suggest positive correlation between capital flow, the real wage rate and migration flow. The estimation of adjustment speeds indicates the positive interaction between those three endogenous variables. Also the findings of our calibration reveal a positive correlation between FDI, the real wage and internal labor mobility in Guangdong (the observed data also reveals positive correlation between FDI, the real wage rate and migration), but relatively low adjustment speeds for \( w \) and \( M \). In this context policies directed towards increasing capital accumulation and relaxing the institutional barriers (or government intervention) have a twofold effect: the adjustment speeds of migration and the real urban wage will be increased. Moreover, the urban real wage and labor absorption capacity will be increased as well.

The projections (Fig 25 - 33) both within and out of sample clearly reflect the fact that the data primarily cover a period of immigration and hence in terms of the phase diagrams we are in a relatively high real wage zone with a relatively low initial stock of migrants. So generally the projections are of further capital growth and immigration. It would be interesting to repeat the empirical analysis for longer samples experiencing emigration and/or immobility.

2.5 Conclusion

We developed a model to look at the simultaneous interactions between three endogenous variables (FDI, real wage and net migration). For our theoretical framework, we start from a benchmark model in the short run, in which the employment is determined by a fixed real wage rate and fixed capital input in the representative firm’s profit maximization process. Net migration is then determined by the expected income gap between regions. Based on
Figure 26: Calibrated capital 2

Figure 27: Calibrated capital 3
Adjustment speeds for net migration

Figure 28: Calibrated migration 1

Adjustment speeds for net migration flow

Figure 29: Calibrated migration 2
Figure 30: Calibrated migration 3

Figure 31: Calibrated wage rate 1
Figure 32: Calibrated wage rate 2

Figure 33: Calibrated wage rate 3
that, we extend the time period to the medium run in which the real wage is flexible, and adjusts according to the partial equilibrium of labor demand and labor supply in the host country \( i \). Finally, we allow capital (FDI) to adjust dynamically following the time path of the marginal product of capital in country \( i \) and the world interest rate at time \( t \). Our migration theory not only includes the determinants of one way migration (immigration) found in the existing literature, but also considers two way migration (immigration and return migration) and labour immobility arising from the migration cost. Importantly, our chapter is the first to recognize that the moving cost causes inherent regime shifts between immigration, return migration and immobility in the dynamic migration process. The non-smoothness and non-differentiable properties of solution paths mean that we extend the standard methods of local stability analysis to take accounts of these special features. This approach should have wider applicability than the present context; in particular any market with a similar switching cost should exhibit that same possibility of immobility and regime shift, thus necessitating the local stability methods used here. The elasticity of labor demand plays an important role in immigrants' extensive margin decisions, in the global dynamics and in the local stability conditions. The stationary states for the three dimensional system exist when the capital plane intersects with the employment plane within the immobility region. When they do exist, there is an infinite number of stationary states lying in the immobility region and along the intersection of the equilibria in the labour and capital markets.

For our empirical work, the scatter diagram (Figure 21) indicates potentially positive simultaneous interactions between the three endogenous variables. Regions with high levels of capital stock and high real urban wage rates (the Pearl River delta cities especially city 1 and 2) normally have a high number of net migrants. While the labor intensive and low urban real wage rate cities (the northern mountain and the southern agricultural regions) have a small number of net migrants. The dynamic adjustment speeds in different variables are positively related. The places with high capital flow adjustment speeds have high values for migration adjustment and relatively fast real wage adjustment. The backward places with slow capital adjustment speeds normally show low values for the adjustment speeds of the migration and real wage. Compared with the adjustment speeds for capital, the overall relatively small value for the real wage and the migration adjustment speeds in Guangdong can be explained by institutional barriers such as the hukou system and the government interventions. These suggest that Guangdong is still in the transition process to a market system and moving barriers especially the hukou system constrain labour mobility. Also we find that the capital market in Guangdong has not been negatively affected by the 1997 Asian financial crisis. This result is consistent with current results supported by the research on the impact of FDI in China (Pan, 2003).
2.6 Some further thoughts

The novelties of our theoretical and empirical work suggest some further studies. We only consider the dynamic interactions between labour and capital markets in the destination. We could think about the dynamics in origin as well. As we allow return migration in our framework, the wage rate in origin should be influenced by return migration as well. If the migrants bring some wealth and send remittances back, the capital market in origin will receive positive inflow. More jobs can be created. People may get job in the origin and stay there rather than migrating to the destination.

Also, theoretically we applied a market adjustment model with a variable wage rate, investment flows and migration. In our model, we have a mechanism for determining not only equilibrium price (wage rate) but also equilibrium quantity (migration and capital flows). The dynamic model developed in this chapter has a disequilibrium process in quantity and price. The structure of the equations is dictated by economic theory and gives us some empirical indications of the time paths of the variables. We find cycles in net migration with extended peaks and troughs (figure 19\textsuperscript{35}). With immobility, capital stock and the real wage rate converge towards a stationary equilibrium conditional on the fixed net migration level. Capital and wage dynamics within the immobility region may lead to a crossing of the migration threshold inducing either immigration or return migration. There is a potentially interesting contrast with a pure VAR time series approach and we come back to this subsequently (Chapter 6).

\textsuperscript{35}Figure 11 portrays the generic cyclical pattern of the variables (from the upper middle immobility region into the return migration region, crossing into immigration region and back to the middle immobility region) in the three dimensional space. And the continued generic cyclical pattern is also captured by figure 11 after the vertical dashed-line.
3 Where is the grass greener? A micro-founded model of migration with application to Guangdong

3.1 Introduction

Associated with chapter 2, chapter 3 considers income disparities as well. It also tries to analyse economic inequality and social inequality in other dimensions, trying to answer question 3 of the Chapter 1. This chapter extends movement between two regions to multiple locations. Individuals are still assumed to be utility maximizers facing pull and push factors. But they are attracted by extra pull factors than just expected high incomes. We follow previous research to identify those other factors, such as an expected income gain (Todaro, 1969; Harris and Todaro, 1970; Johnson, 2003), improved employment opportunities (Greenwood, 1986; Pirciog et al., 2005; Fan, 1996), benefits from the infrastructure of an alternative location (both economic and cultural infrastructure/amenities) (Rappaport, 2008; Chen et al., 2008), and marriage opportunities (Frutado and Theodoropoulos, 2008; Seeborg et al., 2000; Fan and Huang, 1998). An individual may of course be motivated by more than one of these push-pull factors and different possible locations may provide different mixes of these factors. The economic costs to migration include the immediate transport cost but also other fixed and variable costs in the destination location like education for children (Plantinga et al., 2012; Meyerhoefer and Chen 2011), housing market conditions (Vermeulen and Ommeren, 2009), institutional barriers like the hukou in China (Chan and Zhang, 1999, Whalley and Zhang, 2007). Typically the costs will vary with destination but, (apart from the immediate transport cost) not with the original location. The combination of all these forces applied to each individual will determine who moves where and can be put in a general framework in which each possible location is perceived by an individual to have costs and benefits (Krugman, 1992).

In a matching equilibrium, the population distribution between locations and the specific features of different locations are such that no individual has an incentive to move. Matching theory results by Ekeland et al (2004) and Heckman et al (2002) in particular are relevant to our purpose in which heterogeneous individuals are sorted into different outcomes by a market system which allows for pricing by type of individual. Our theoretical approach applies some of the insights of this literature to the migration process, recognising that migration is essentially a disequilibrium event.

The aim of this chapter is to develop a theoretical framework\textsuperscript{36} which allows for a menu of alternative migration pull factors in a geographical domain with multiple possible destinations. Each individual currently resides in a

\textsuperscript{36}To our knowledge, our work is the first to aggregate migration flows from individual decisions allowing for multiple pull factors and multiple locations. Bazzi (2012) develops a microfounded model of aggregate migration flows to study to what extent financial barriers limit international migration. But he did not consider multiple pull factors and multiple location choices.
particular location and has to weigh up the alternative menu of factors available in each different location, together with the costs of moving to that location. The outcome is a migration decision for each individual and, aggregating these over individuals in each current location, we derive the net migration flows between locations. Apart from the theoretical insights gained into modelling multiple motives for migration and multiple locations, our second main aim is to use this framework to generate an empirically applicable migration equation system to aggregate data. We then apply this to econometric analysis of intra-province migration patterns in Guangdong, one of the provinces of China with the highest such migration rate.

Our main theoretical result is that the interplay of multiple motives and multiple possible destinations for migration will lead to agglomeration. Individuals will typically want to move to a location seen as best for the combinations of its advantages. However individuals already living in such a favoured location, but whose individual circumstances have turned out poorly there, may wish to leave and move to the location offering the second best advantages. Our approach establishes these tendencies in quite a general framework. We add some assumptions to the framework which results in a net migration equation which can be empirically applied in a setting with multiple migration motives and multiple locations. We apply this to intra-province aggregate migration flows in Guangdong for the period 1990-1999 with 18\textsuperscript{37} locations and 4 migration motives. We find that the approach represents the data quite closely and that there is a high degree of individual preference homogeneity within different locations. This implies that if moving costs are low, the equilibrium spatial population distribution should yield equal utilities between locations in the factors which cause migration. However this is not the case for all factors, in fact regional inequality has been growing for these despite the high net migration flows. We conclude that spatial equilibrium in different factors in Guangdong has not yet been reached.

Section 2 briefly reviews existing knowledge of factors causing migration and empirical work on Guangdong, section 3 develops the theoretical framework, section 4 describes the data, outlines the econometric strategy and presents the empirical results. Section 5 contains a conclusion.

3.2 Literature review

Some of the comprehensive surveys of migration research recognize the multiple push-pull factors which drive it and that migrants face a choice between alternative destinations. Greenwood (1997) studies the determinants of migration in developed countries, including characteristics both of places and of individuals and households. Taylor and Martin (2001) study the complexity of migration determinants and its impact in rural economies. In the context of climate driven migration, Lilleor and Broeck (2011) also recognize the variety of migration causes.

\textsuperscript{37}In this chapter, we did not drop the two aggregated regions, this leaves us 18 cities.
Widely noted pull factors that we subsequently use are:

(i) Expected Wage Income.

In the classic Harris-Todaro model approach, migrants are motivated by the high wage and the high employment probabilities in the chosen destination. There are many subsequent applications of these two ideas, for example Johnson (2003) finds that rural-urban migrants in China move with an urban-rural wage gap of 50 – 70%. For employment probability, Greenwood et al (1986) find an elasticity of about 0.5 of migrants to job vacancies, and also unemployment shocks often hit immigrant workers most heavily (Brucker and Jahn, 2008).

(ii) Expected Self-Employment Profits.

In European countries, 10-25% of migrants establish themselves as self-employed in the destination (OECD, 2010) and in China it rises to 40% or more (RUMIC, 2007) or even 60% in Guangdong (Fan, 1996, 1999, 2003). There is evidence that self-employed migrants are most influenced by the size of market and population of the destination (Kugler and Rapoport, 2005; Federici and Giammetti, 2010). In the UK there is a high concentration of self-employed immigrants in London (Dustmann et al., 2007) and 47% of the national self-employed migrants live there (The Migration Observatory, 2012).

(iii) Location Infrastructure.

Available local public goods differ by location eg transport and communications, public health or education services, cultural aspects and these affect the perceived quality of life in locations (Rappport, 2008). Synergies and externalities have a similar effect (Chen et al., 2008). Often these come just from the size of the population in the destination.

(iv) Female Migration & Marriage Motives.

Globally around 50% of migrants are female (Piper, 2005). Marriage is one important pull factor in the UK (accounting for 40% of migrant settlements during 2008-2010 (Charsley and Liversage, 2011) and Asia (Fan and Huang, 1998; Zhu, 2002). Another is employment opportunities which may be gender specific eg in textile industries (Seeborg et al, 2000). Marriage can also interact with employment prospects (Frutado and Theodoropoulos, 2008).

An important issue is identifying and modelling the costs associated with migration between particular locations. Transport costs depend on the physical distance between locations (Poncet, 2006). In several NELM (New Economics of Labour Migration) models (Mesnards, 2000; Docquier and Rappoport, 2008), fixed costs of migration are used. Institutional and regulatory barriers such as visas or the hukou system in China (Chan and Zhang, 1999) create costs. Whalley and Zhang (2007) conclude that inter-province wage differentials caused by the hukou system impose a significant welfare loss. In Guangdong (our focus for empirical work) the majority of migrants initially
hold an agricultural hukou (Fan, 1996, 1999, 2003; Zhu et al., 2009) which means permanent migration will require a change of hukou.

3.3 Theoretical background

3.3.1 General Framework

There are \( n \) individuals \( h = 1..n \) who are each initially located in one of \( m \) different places \( i = 1..m \). Each individual derives utility from \( K \) (continuously divisible) location specific factors \(^{38} f = (f_1, f_2..f_K) \). Individuals have utility \( u(f) \) and by definition utility is increasing in each factor. Within each location there is a multivariate distribution of the factors across the individuals who reside there, so for example individuals \( h \) and \( l \) who both reside in the same location may respectively enjoy values of the factors \( f = (f_{1h}, f_{2h}..f_{Kh}) \) and \( f = (f_{1l}, f_{2l}..f_{Kl}) \). This generates intra-location differences. In addition locations differ in the distribution of factors so there are also inter-location differences, stressing the role of individual heterogeneity in line with Borjas,(1999) and Ekeland et al (2004). For example the mean or the variance of factors realised may differ between locations (one location may be relatively rural with a low mean and variance of wages, another have a more mixed industrial structure with both a higher mean and a higher variance in wages). In terms of matching theory there is bunching in locations because two individuals who are both ex ante best suited to a particular location experience different "luck" in accessing the attributes of the location eg by chance one individual may get a better job offer than the other even though they are of identical productivity. But on average one location may attract mainly low skilled workers whilst another attracts high skilled workers. Thus within location utility differences largely result from luck but between location differences from more deterministic heterogeneity in location and individual characteristics.

In a matching model, Ekeland et al (2004) have derived a closed form solution for the equilibrium matching allocation when individual utility is linear in unobserved deterministic heterogeneity but quadratic in the location factors. Suppose that average utility derived from factors \( z \) is quadratic in \( z, a'z + 1/2z'Bz \), and immigrants into the overall area have to choose their location. Individual deterministic heterogeneity, \( \varepsilon \), causes differences in realised utility between individuals who choose the same location according to \( \varepsilon a'z + 1/2z'Bz \). So the \( N \) locations can be ranked by each individual with the best location for \( \varepsilon \) being \( \max_{i \in N} \varepsilon a'z^i + 1/2z'^iBz^i \) where \( z^i \) is the vector of factors in location \( i \).

Fig 34 shows the utility available to different individuals from locating in each of four different locations \( i = 1,..4 \). Locations 3 and 4 are dominated and optimal for no individual. Any individual with \( \varepsilon > \varepsilon^* \) has location 1 as their

\(^{38}\)The idea is that the factors measure variables like the wage, employment opportunities, infrastructure and local public goods, profit prospects for self employed individuals, marriage prospects, etc.  
\(^{39}\)In fact in this special case where the matching is one-sided, if the realised utility has the form \( \varepsilon f(z) + g(z) \), where \( f(\cdot) > 0 \) the same argument follows.
top ranked destination with second best location 2. Any individual with $\varepsilon < \varepsilon^*$ has top ranked location 2 with second best choice either location 3 or location 1. Hence the population is sorted into one group who will migrate into location 1 and another who will migrate into location 2. If that is the whole story then each individual will locate in his best destination.

We have to adapt this to analyse movement between locations eg from one location to another within Guangdong. Each person within any given location compares their current circumstances with what they can expect to attain by a move to an alternative location, this interaction determines who moves where. An individual $h$ currently in $i$ could move to location $j$. If he moves, he does not know exactly what factor combination he will get in $j$ because there is heterogeneity within a location in individual experience there. He has to assess the range of payoffs he could get from different possible combinations $x$ of the factors $f_{j1}, f_{j2}, \ldots f_{jK}$ which might occur in $j$. We assume each individual assesses his gross benefit from moving to $j$ as $Eu(f|j) = \Sigma_x \pi^j(f_{j1}, f_{j2}, \ldots f_{jK})u(f_{j1}, f_{j2}, \ldots f_{jK})$. Here $\pi(.)$ has the interpretation that in $j$, $h$ has the chance $\pi$ of getting the factor combination $f_{j1}, f_{j2}, \ldots f_{jK}$ in $j$.

It is then natural for any individual to condition his probability distribution over factors in $j$ on the mean level of the factors there (which are observable to the individual, $\mu_1, \ldots \mu_K$), so that $\pi^j = \pi(f_{j1}, f_{j2}, \ldots f_{jK}, \mu_1, \ldots \mu_K)$.

Again locations are ranked by each individual, but now the best off individuals in any location may prefer to stay where they are rather than to move, whilst those with the lowest standard of living in a location are the most likely to move. The heterogeneity within a location may partly depend on deterministic individual characteristics but also in large measure on differences in luck that individuals have experienced in the current location. We can model this by assuming that any individual who enters a new location has an equal chance of enjoying the mean

Figure 34: Available utility levels for individuals and locations
standard of living there. They can also stay in their present location and enjoy their current standard. The first diagram of Figure 35 shows the utility distribution of different individuals in a given location as a function of their past experience (luck) and type on the horizontal axis, for example ranging from very unlucky to very lucky or from unskilled to skilled. The positively sloped lines show the current utility enjoyed from different types at present in that location. The horizontals $E_1, E_2$ show the mean utility level that any type of one location can expect from a move to the other location. Thus in location 1 all types with utility below currently below $\varepsilon_1$ will move to location 2 but all others will remain in location 1. Similarly all types in locations 2 with $\varepsilon < \varepsilon_2$ will move to location 1 but all others will remain. In fact in our data observed migrants actually have very similar individual characteristics in terms of age, education, marital status (RUMICI, 2007) so presumably the within location heterogeneity is largely caused by past experience. Also in the data the observed city factors $z$ in the data actually have strong dominance relations (the same city empirically tends to come top on each factor in our sample period). This gives us the second diagram (Fig 35) in which the utility distributions of different locations never intersect and the mean utility of each location is unambiguously ranked. In this case below the best location, the worse off in each location all move into location 1, but the worst off in location 1 move to the second best location 2.

The implication is that with homogenous expected utility (preferences $u(\cdot)$ and probabilities $\pi_{x}^{j}$) between individuals, all individuals in all locations will agree on $j^{\ast}$ as being the location offering the highest $Eu(f|j)$ . Any two individuals who presently have the same factors will agree on the new particular $j^{\ast}$ location that offers the best standard of living $Eu(f|j^{\ast})$ irrespective of where they are currently located (so long as they are not located in $j^{\ast}$).

However measured in commensurate utility terms, if the individual does move from $i$ to $j$ he also bears costs
\(v_{ij}\) so the net expected benefit he could secure from a move to \(j\) is

\[
NB_{jh} = \sum_x \pi^j_x u(f^j_{1x}, f^j_{2x}, \ldots, f^j_{Kx}) - v_{ij} - u(f_{1h}, f_{2h}, \ldots, f_{Kh})
\]

The best alternative location for \(h\) is then \(NB_{j^*h|i} = \max_j \{NB_{jh}|j = 1\ldots n|i\}\) and \(h\) will move to \(j^*\) if \(NB_{j^*h|i} > 0\).

It is also natural to assume that the moving cost \(v_{ij}\) is additive in \(ij\), that is \(v_{ij} = c_i + c_j\). Any moving costs satisfy this. There is little reason to suppose that leaving costs from exiting \(i\) depend on the destination so \(v_{ij} - v_{ik} = v_{ij} - v_{ik}\). With an appropriate choice of units, this implies that \(v_{ij}\) is additive. Indeed the exit costs from a location may be very close to zero. With additivity of costs, the best destination \(j^* = \max_j \{\sum_x \pi^j_x u(f^j_{1x}, f^j_{2x}, \ldots, f^j_{Kx}) - c_j\}\). If \(v_{ij}\) is small too, then essentially the utility differential \(Eu(f^j) - u(f_{1h}, f_{2h}, \ldots, f_{Kh})\) will determine the best destination for individual \(h\).

If an individual is currently in \(j^*\) any uncertainty about the combination of factors he faces there has already been resolved. However such an individual can still think about moving elsewhere and if he is currently low down in the distribution of attainable utilities in \(j^*\), then he may have the prospect of a higher expected utility by moving to an alternative location. This location \(j^{**}\) will be the one which is judged second best by all individuals. That is \(j^{**}\) maximizes \(\sum_x \pi^j_x u(f^j_{1x}, f^j_{2x}, \ldots, f^j_{Kx}) - c_j\) over all \(j\) other than \(j^*\).

How does this general framework lead to migration flows between locations? In any location \(i\) there is a distribution of the factors \(f\) defined by the cdf \(G_i(f)\) with associated density \(g_i(f)\). Define the lower sets of \(u(.)\) by \(L(u^*) = \{f|u(f) \leq u^*\}\). Then there is a corresponding distribution of utilities in the location of \(H_i(u^*) = \Pr(u(f) \leq u^*) = \int_{f \in L(u^*)} g_i(f)df\). All individuals in location \(i\) with \(\sum_x \pi_x (\mu^j_1, \ldots, \mu^j_K) u_x - c_j > u(f^j_{1j} \ldots f^j_{Kj}) + c_i\) will desire to move. If the exit cost \(c_i\) is zero, a proportion \(H_i(\sum_x \pi_x (\mu^j_1, \ldots, \mu^j_K) u_x - c_j)\) could increase their utility by a move to location \(j\). With \(N_i\) individuals currently in location \(i\), the number who could benefit from migrating from \(i\) to \(j\) will be \(N_i H_i(\sum_x \pi_x (\mu^j_1, \ldots, \mu^j_K) u_x - c_j)\). However since all individuals not presently in \(j^*\) agree that \(j^*\) offers the highest gain from all possible moves, the migration flow into locations other than \(j^*\) will be zero from origins other than \(j^*\). So we derive aggregate immigration into \(j^*\) from all other locations of \(I_j = \sum_{i \neq j} N_i H_i(\sum_x \pi_x (\mu^j_1, \ldots, \mu^j_K) u_x - c_j)\) and zero immigration into any other location.

However some individuals at present in \(j^*\) will have had unfortunate experiences there and will be low down in the utility distribution in \(j^*\). They could gain from a move to the second best location. The number of individuals in \(j^*\) who see they can secure the highest improvement in their standard of living by a move to \(j^{**}\) is \(N_{j^*} H_{j^*}(\sum_x \pi_x (\mu^{j^{**}}_1, \ldots, \mu^{j^{**}}_K) u_x - c_{j^{**}})\).

**Proposition 8** If individual preferences and chances of success in any destination location are common, and if the
migration cost is additive with exit costs being very low relative to entry costs, migration flows will be out of all but two locations. The two locations with immigration will be ranked as the top two in terms of the overall standard of living.

This is a general abstract formulation and the idea of common preferences for all individuals is very restrictive. However, we find this proposition matches our empirical data which is characterised by agglomeration.

One natural specialization that we will use for empirical work is to assume that $H_i$ is linear

$$H_i(x_j) = \sum_k (a_k + b_k x_j^k) - c_j.$$ 

It is obviously very convenient and can arise in several scenarios:

(i) Specialized preferences

Suppose that in any location the population partitions into groups who are each affected by a single different factor. For example one group is employees who are motivated solely by labour earnings, another group are disabled individuals who are heavily dependent on local public goods like health care. Each individual within the group for which the $k$th factor is the sole determinant of utility judges alternative locations solely in terms of that factor. All individuals in this group in any location judge the particular location

$$j^*(k) = \arg \max_j \sum \pi_x (\mu^j_k) u(f_{kx})$$

as the most desirable in which to live. There is a distribution of the $k$th factor within location $i$ which generates a utility distribution for this group of $H_{ik}(u(f_k))$. From each location $i$ a proportion of individuals $H_{ik}(\sum \pi_x (\mu^j_k) u(f_{kx}))$ in the $k$-group will want to move to $j^*(k)$.

Taking each group in turn, for each factor there may be a different top ranked location $j^*(k)$ and a particular location may be top ranked on more than one factor. The immigration into any location $j$ will be the sum of those individuals $k$ in other locations $i$ who judge $j$ as the best destination on any one of the $K$ factors. So we get immigration into $j$ of $
\sum \pi_x \sum_k N_{ik} H_{ik}(\sum \pi_x (\mu^j_k) u(f_{kx}) - c_j) \quad \text{where } N_{ik} \text{ is the number of individuals living in } i \text{ who are motivated only by the } k \text{th factor.}$

If the distribution of the factor within groups is uniform and so the cdf is linear, this reduces to

$$\sum \pi_x \sum_k N_{ik} (a_k + b_k x_j^k) - c_j.$$ 

In addition there will be the second choice individuals who are presently in the top location for a particular factor that determines their standard of living, but whose present state on that factor is so unfavorable that they would be better off moving to the second best location on that factor.

(ii) Complementary preferences:

If $u(f_{1k}, f_{2k}, \ldots, f_{Kk}) = \min(f_{1k}, f_{2k}, \ldots, f_{Kk})$ then effectively each individual is constrained by a single factor (ignoring ties) in their current position. The utility they anticipate from any alternative location is

$$\sum \pi_x \min(f_{1x}, f_{2x}, \ldots, f_{Kx}).$$

We can partition the support of the distribution of $f$ into regions $R_k$ in which the $k$th factor is critical (for simplicity in the argument ignoring ties, which have minimal probability) so that eg for any $(f_{1x}, f_{2x}, \ldots, f_{Kx}) \in R_k$, $f_{kx}^j = \min(f_{1x}, f_{2x}, \ldots, f_{Kx})$. If $\Pi^j_k$ is the probability of the region of the support in which the $k$th factor is
the constraining factor on utility, we can write $\Sigma_x \pi^j_x \min(f^j_1, f^j_2, \ldots f^j_K) = \Sigma_n H^j_n \Sigma_k \pi_{k|x} f^j_k$ where $\pi_{k|x}$ is the probability of particular values of the $k$th factor given that this factor is the minimal constraining one. For example an individual is affected by all factors but needs both a high wage and good infrastructure in fixed ratios. If the ratio is not met then his standard of living is set by the lower of the two. Hence his expected utility is determined by the chance that each of the two factors is constraining, and then within the region where one factor is critical, the expected value of the utility of that critical factor.

Under the assumption that $H_i(\Sigma_x \pi_x (\mu_1^j, \ldots \mu_K^j))u_x - c_j = \Sigma_k (a_k + b_k \mu_k^j) - c_j$, aggregate immigration into $j^*$ from all other locations is

$$M_{j^*} = \Sigma_{i \neq j^*} N_i H_i(\Sigma_x \pi_x (\mu_1^j, \ldots \mu_K^j))u_x - c_j^*) = \Sigma_{i \neq j^*} N_i (\Sigma_k (a_k + b_k \mu_k^j) - c_j^*)$$

However if $j^{**}$ is the second best location ($j^{**} = \arg \max (\Sigma_x \pi_x (\mu_1^{j^*}, \ldots \mu_K^{j^*}))u_x - c_j^* | j \neq j^*$) then

$$N_{j^*} H_{j^*}(\Sigma_x \pi_x (\mu_1^{j^{**}}, \ldots \mu_K^{j^{**}}))u_x - c_j^{**}) = N_{j^*} (\Sigma_k (a_k + b_k \mu_k^{j^{**}}) - c_j^{**})$$

will leave the top location $j^*$ seeking better fortune in $j^{**}$. Hence we can write the full set of net migration flows between locations as

$$NM_{j^*} = \Sigma_{i \neq j^*} N_i (\Sigma_k (a_k + b_k \mu_k^j) - c_j) - N_{j^*} (\Sigma_k (a_k + b_k \mu_k^{j^{**}}) - c_j^{**})$$

$$NM_{j^{**}} = N_{j^*} (\Sigma_k (a_k + b_k \mu_k^{j^{**}}) - c_j^{**}) - N_{j^{**}} (\Sigma_k (a_k + b_k \mu_k^{j^*}) - c_j^*)$$

$$NM_i = -N_i (\Sigma_k (a_k + b_k \mu_k^j) - c_j) \text{ for } i \neq j^*, j^{**}$$

### 3.4 An Empirical Application to Guandong

China’s rapid development has been largely regional with construction of infrastructure and establishment and growth of an industrial base concentrated in particular areas. Resulting regional inequalities have stimulated migration, although the hukou system has acted as a migration barrier of variable force. Guangdong is a Chinese province close to Macao and Hong Kong, which has attracted government financial incentives for development and high FDI from Hong Kong. Growth has concentrated around the Pearl River, triggering high levels of intra-province migration (2.7 times higher than its already high inter-province immigration). We use 18 Guangdong city areas as our location units\(^{40}\) for the period 1990-1999. The cities are heterogeneous: Guangzhou (1), Shenzhen (2), Zuhai (3), Dongguan (10), Zhongshan (11) and Foshan (13) are industrialised Pearl River cities. Dongguan (10), Zhongshan (11) and Foshan (13) are traditional industrial centres and Shenzhen (2) and Foshan (13) are also

\(^{40}\)These are an aggregation of the 21 administrative city areas to ensure unique boundaries over 1990-1999. Each city area has both an urban and rural part.

The data is taken from Guangdong Statistical Yearbooks, 1990-1999.
Figure 36: Map of Guangdong

Special Economic Zones (Aten, 1979) with government incentives. The cities further from the Pearl River nexus, Shaoguang (5), Heyuan (6), Meizhou (7), Huizhou (8) and Shanwei (9) in the north, Zhanjian (15), MaoMing (16), Yangjiang (14) and Qiangyuan (18) in the south, show lower net migration and some have net emigration. Finally there are cities (22) and (23) formed by merging the administrative units of (4), (19), (20) and (17), (21) (Figure 36). The merged city has very low net migration.

For each city and year the variables we measure are

(i) permanent net migration generally involving a change in hukou ($NM$),
(ii) the population (both genders and all hukou types) ($P$)
(ii) urban employment ($E$) and urban wage ($W_u$) in the top three permanent sectors (state owned, urban collective owned and other units)
(iii) rural income per capita ($W_r$) (defined as the ratio of gross agricultural output in rural primary industry to rural primary industry employment. Labour is the dominant agricultural input)
(iv) a city specific consumer price index $\Pi$ used to derive the real urban and rural wages $w_u = W_u/\Pi, w_r = W_r/\Pi$
(v) capital stock $K$ derived from an initial stock, foreign direct investment flows, and a city specific depreciation rate $^{42}$.  

$41$ Detailed definitions are in the appendix.

$42$ $K_t = (1 - \delta)K_{t-1} + FDI_t$, where $\delta$ is the depreciation rate. The base value of capital stock is given by the 1992 historic cost value.
During 1990-1999, Guangdong enjoys all the priorities such as free tax rate for foreign capital inflow and free trading zones. These promote the economic development in Guangdong. Local residents benefit from the rapid economic development and are rarely attracted to emigrate. People living outside of Guangdong are motivated to immigrate in but face very high entry barriers (e.g. hukou, high living cost, job discrimination for outside job seekers). The high entry barriers prevent a large volume of external migration from outside of Guangdong and protect free movement within Guangdong. Given this empirical evidence, we use a city panel data for internal migration within Guangdong over 1990-1999 to match the framework considering a closed system.

Table 3 describes the different city characteristics: the net migration population ratio \( (NM/P) \), \( w_u, w_r \), urban hukou holders as a % of the city population \( (Urbanhukou/P) \), the late marriage rate\(^{43}\), capital stock per city inhabitant \( (K/P) \), the number of single females, the population (both in millions of people) and the city size in million square metres. The first three cities are the key Pearl delta cities and are the most urbanized, physically small, highly industrialized and with a high population density, the highest urban wage but quite high urban-rural real wage inequality. Together with Dongguan they share the highest capital/population ratio. Proportionally to the population, the capital Guangzhou, Zhuhai and especially Shenzhen have a low prevalence of single females and they also have a high late marriage rate, indicating both a better educated and slightly older population. At the other extreme, cities 14-16 have low degrees of urbanization and relatively low real urban wages and real city income although the rural wage is not very low. The highest inward net migration is into the Pearl River cities 1-3 and Huizhou and Foshan. The city with the highest outward net migration is Shaoguan.

Table 4 shows the coefficient of variation across cities of the variable in question through time. It indicates growing inequality between the cities over time in both rural and urban wages and in capital stock per inhabitant. Interestingly variations in the late marriage rate between cities is falling but variations in the number of single females between cities and size differences between cities are roughly constant. Urbanization seems to be spreading slowly across cities so that variations between cities are gently falling. In a word overall, the rapid development since 1995 has generally been accompanied by an increase in inequality between city areas.

### 3.4.1 Applying the Theory to Guangdong

Guided by earlier studies and the descriptive statistics above, we select four factors as motives for migration:

(a) to work as an employee in the three chief employing organizations\(^{44}\) in \( j \), in which case the primary motivation of assets. The depreciation rate is computed as the % difference between the net value of fixed assets and the historic value of fixed assets in 1992, the mean of this is about 25%.

\(^{43}\)Defined as the number of females who were at least 23 years old at marriage as a proportion of the total number of first marriages.

\(^{44}\)These are either state or urban collective units, or private sector units with joint ownership, shareholding or foreign ownership (ie excluding self employment).
### Table 3: Average value of key variables

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<th>City</th>
<th>N/P</th>
<th>Real Wu (yuan)</th>
<th>Real Wr (yuan)</th>
<th>Urban Hukou/P</th>
<th>Late marriage rate(%)</th>
<th>K/P(1000 yuan)</th>
<th>Single Female (million)</th>
<th>Population (million)</th>
<th>Urban population (m sq)</th>
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<td>Mergedcity22</td>
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<td>1.37</td>
<td>0.88</td>
<td>0.23</td>
<td>72.36</td>
<td>1.88</td>
<td>3.44</td>
<td>11.0</td>
<td>10.26</td>
</tr>
<tr>
<td>Mergedcity23</td>
<td>0.007</td>
<td>1.37</td>
<td>0.99</td>
<td>0.24</td>
<td>57.39</td>
<td>1.59</td>
<td>1.82</td>
<td>5.9</td>
<td>22.84</td>
</tr>
</tbody>
</table>

### Table 4: Inequality between cities over time (coefficient of variation)

<table>
<thead>
<tr>
<th>Year</th>
<th>N/P</th>
<th>Real Wu (yuan)</th>
<th>Real Wr (yuan)</th>
<th>Urban Hukou/P</th>
<th>Late marriage rate(%)</th>
<th>K/P(1000 yuan)</th>
<th>Single Female (million)</th>
<th>Population (million)</th>
<th>Urban population (m sq)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>1.50</td>
<td>0.00</td>
<td>0.00</td>
<td>0.56</td>
<td>1.29</td>
<td>0.68</td>
<td>0.69</td>
<td>0.69</td>
<td>0.69</td>
</tr>
<tr>
<td>1991</td>
<td>0.04</td>
<td>0.10</td>
<td>0.56</td>
<td>0.22</td>
<td>1.36</td>
<td>0.68</td>
<td>0.67</td>
<td>0.70</td>
<td>0.70</td>
</tr>
<tr>
<td>1992</td>
<td>0.08</td>
<td>0.11</td>
<td>0.57</td>
<td>0.19</td>
<td>1.42</td>
<td>0.67</td>
<td>0.70</td>
<td>0.70</td>
<td>0.70</td>
</tr>
<tr>
<td>1993</td>
<td>1.85</td>
<td>0.09</td>
<td>0.18</td>
<td>0.54</td>
<td>1.45</td>
<td>0.68</td>
<td>0.68</td>
<td>0.68</td>
<td>0.68</td>
</tr>
<tr>
<td>1994</td>
<td>2.01</td>
<td>0.07</td>
<td>0.19</td>
<td>0.53</td>
<td>1.51</td>
<td>0.69</td>
<td>0.69</td>
<td>0.69</td>
<td>0.69</td>
</tr>
<tr>
<td>1995</td>
<td>1.95</td>
<td>0.08</td>
<td>0.20</td>
<td>0.53</td>
<td>1.54</td>
<td>0.66</td>
<td>0.67</td>
<td>0.67</td>
<td>0.67</td>
</tr>
<tr>
<td>1996</td>
<td>1.47</td>
<td>0.11</td>
<td>0.33</td>
<td>0.53</td>
<td>1.25</td>
<td>0.69</td>
<td>0.69</td>
<td>0.69</td>
<td>0.69</td>
</tr>
<tr>
<td>1997</td>
<td>1.64</td>
<td>0.13</td>
<td>0.24</td>
<td>0.51</td>
<td>1.58</td>
<td>0.69</td>
<td>0.70</td>
<td>0.70</td>
<td>0.70</td>
</tr>
<tr>
<td>1998</td>
<td>1.72</td>
<td>0.16</td>
<td>0.26</td>
<td>0.52</td>
<td>1.65</td>
<td>0.69</td>
<td>0.69</td>
<td>0.69</td>
<td>0.69</td>
</tr>
<tr>
<td>1999</td>
<td>1.87</td>
<td>0.17</td>
<td>0.28</td>
<td>0.51</td>
<td>1.62</td>
<td>0.71</td>
<td>0.72</td>
<td>0.72</td>
<td>0.72</td>
</tr>
</tbody>
</table>
is an expected real wage difference between cities.

(b) to be self-employed in \( j \), where the difference in profit opportunities between \( i \) and \( j \) accounts for the move. The size of the market \( P \) is one proxy measure, capital intensity \( K/P \) is another. These variables should reflect the demand for services of the self-employed and their cost determining variables.

(c) to get married/join friends etc who are in \( j \). One measure of the relative desirability of different cities in their marriage opportunities is given by the gender structure of the population of single individuals in different cities. We measure this by the number of single females.

(d) to leave a mainly agricultural city area to move to a more urbanized city area where infrastructure is better developed, measured by the \% of the urban population holding an urban hukou.

We model the city specific migration costs by a mix of a distance measure (a city specific constant \( A_j \)) and the \% of the urban population holding an urban hukou. The latter reflects the severity with which the hukou policy is applied in different areas, a high ratio indicates a greater barrier. Note we are using the urban hukou ratio to reflect two different and opposite signed effects on net migration.

Our theory as exposited in (7) determines net migration flows from the wage, self employment profit, marriage chance and urban infrastructure gaps between cities together with the migration costs

\[
NM_{j^*} = A_{j^*} + \sum_{i \neq j^*} N_i (\Sigma_k (a_k + b_k \mu_k^j) - c_j) - N_{j^*} (\Sigma_k (a_k + b_k \mu_k^{j^*}) - c_{j^*})
\]

\[
NM_{j^{**}} = A_{j^{**}} + N_{j^*} (\Sigma_k (a_k + b_k \mu_k^{j^*}) - c_{j^*}) - N_{j^{**}} (\Sigma_k (a_k + b_k \mu_k^{j^{**}}) - c_{j^{**}})
\]

\[
NM_i = A_{i} - N_i (\Sigma_k (a_k + b_k \mu_k^i) - c_j) \text{ for } i \neq j^*, j^{**}
\]

This has the interpretation of a gaps model (Zhu, 2002) in a multi area and multivariate context\(^45\). There are 6 coefficients to estimate for each city giving a total of 108 regression coefficients (detailed definition of the dependent variable and regressors is in the appendix).

Having defined the rankings of cities\(^46\) such that the top ranked is the most desirable, and individuals move when possible to the top two ranking cities, the coefficients \( b_k \) should generally be positive. There are some possible ambiguities: differences in the number of single females may proxy the availability of female worker jobs especially in the textile industries (Fan, 2003; Huang, 2001) or may proxy marriage chances. Similarly, depending

\(^45\)Instead of thinking of the distribution of the factor within a city, this can be interpreted as saying for example that net (and gross) migration from a city ranked three or lower into the top city is the total factor gain from the lower rank city achieving the mean factor of the top city.

\(^46\)The ranking of cities on different factors may vary with time. Broadly the Pearl delta cities (1,2,3), Foshan and city 22 are generally ranked in the top two on most factors. The implication is that we should expect to see intra-Guangdong emigration from the remaining thirteen cities which are never ranked in the top two on any criterion for migration but inward immigration into the ranked cities.
on whether capital and labour are substitutes or complements, capital stock can have an ambiguous effect on employment prospects via the demand for labour. Finally, population can have an ambiguous effect, it could reflect disadvantages due to congestion in an area or the level of demand for the services and output of the self employed.

We add a disturbance $\varepsilon_{it}$ which is assumed to have a zero mean at each $i,t$ and initially for given $i$, to be independent over time $t$ with a constant covariance matrix across cities. We test the lack of autocorrelation of the residuals following estimation using Wooldridge’s (2002) panel serial correlation test.

Adding the disturbance and using more succinct notation, (8) becomes

$$NM_{it} = A_i + \sum_{x|i=j} B^x_i Gap^x_{it} + \varepsilon_{it}, E\varepsilon_{it} = 0, E\varepsilon_{it}\varepsilon_{js} = \sigma_{ij} \text{ for all } t,s$$

where (details of the gaps are defined in the appendix).

$$Gap_{it} = \begin{cases} \sum_{j\neq i} \mu_j, N_j - \mu_j, N_j \text{ if } i = j^* \\ \mu_j, N_j - \mu_j, N_j \text{ if } i = j^{**} \\ -\mu_j, N_i \text{ if } i \neq j^*, j^{**} \end{cases}$$

Secondly we have to specify the covariance structure between cities. For each city the variance is constant over time since it is iid. But the variances could differ between cities, effectively giving the disturbances a panel structure. Similarly the shocks of any two cities may be positively correlated (like a common global shock to all cities) or negatively correlated (eg if there is some uncertainty over the best destination for a migrant who has decided to move within Guangdong). We use the Pesaran(2006) and Hoyo and Sarafidis (2006) test for cross section dependence. The test statistic has an approximate normal distribution which should be valid even in small samples.

We estimate the parameters by GLS allowing for the variances of disturbances to differ by city. In order to check the robustness of GLS, we also estimate by weighted OLS and find equivalent results. We allow the constant terms ($A_i$) and all the slope coefficients $B^x_i$ to vary by city through the use of dummy variables for each city. The most general model has 108 regression parameters\(^{47}\), which shows no evidence of serial correlation or cross section dependence and also no evidence of panel effects or heteroscedasticity in the disturbances. So subsequently we use a diagonal covariance matrix of the form $E\varepsilon_{it}\varepsilon_{js} = 0$ for all $t,s$ and $i \neq j$ but $E\varepsilon_{it}^2 = \sigma^2_i$ for all $t$.

We impose zero and equality restrictions on coefficients to reduce the system to 67 coefficients (Tab 5). The restrictions are accepted on a likelihood ratio test, applied sequentially and still has no autocorrelation or cross city correlation (Table 6). Estimating the same model by weighted OLS (allowing for disturbance variances to vary by city) gives very similar coefficients, an $R^2 = .959$ and easily passes a Ramsey Reset test. The weighted OLS

\(^{47}\)In fact we drop one regressor to avoid multicollinearity, leaving 107 coefficients.
residuals also show no sign of autocorrelation or cross section dependence. The evidence is that this base model with 67 coefficients is an adequate specification of the migration process. The accompanying plots show the relation between the actual and predicted net migration by city (Figure 37). Generally the model is replicating the data as one would expect, smoothing some of the sharper fluctuations especially in cities 1, 8, 22.

However many of the coefficients are very similar across cities, although there are some outlying gap-city combinations. We impose zero and equality restrictions on coefficients on the 108 coefficients model to reduce the system to 27 coefficients model. The restrictions are selected according to T and F tests. This leads to a final model with just 27 coefficients (Table 7) with a loglikelihood of −29.60 and the model also passes all the diagnostic tests. Comparing the plots (Figure 37 and Figure 38) of the actual and predicted values by city for the 27 and 67 coefficient models reveals that we lose relatively little in terms of goodness of fit from imposing these restrictions.
Table 6: Diagnostic test statistics

<table>
<thead>
<tr>
<th>Test</th>
<th>GLS</th>
<th>GLS WLS</th>
<th>OLS</th>
<th>OLS WLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>GLS GLS WLS GLS WLS</td>
<td>108</td>
<td>67</td>
<td>67</td>
<td>27</td>
</tr>
<tr>
<td>Wooldridge SR</td>
<td>F(1,17)=4.097, p=0.058</td>
<td>F(1,17)=2.531, p=0.111</td>
<td>F(1,17)=3.55, p=0.081</td>
<td></td>
</tr>
<tr>
<td>Pesaran test</td>
<td>N(0,1)= -1.32, p=0.181</td>
<td>N(0,1)= -0.70, p=0.480</td>
<td>N(0,1)= -0.59, p=0.551</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.959</td>
<td>0.841</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reset test</td>
<td>F(3,73)=0.33, p=0.800</td>
<td>F(3,113)=0.64, p=0.593</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sargan test</td>
<td>$p=0.265$</td>
<td>$p=0.593$</td>
<td>$p=0.265$</td>
<td></td>
</tr>
<tr>
<td>Hausman-Wu test</td>
<td>$p=0.42$</td>
<td></td>
<td>$p=0.42$</td>
<td></td>
</tr>
</tbody>
</table>

Figure 37: GLS 67 coefficients Y vs predicted Y

on the 67 coefficient base model. We still track the data quite well and pick up most turning points in the net migration data. The 27 coefficient model represents our final model. For the sake of robustness we also estimate the final model by weighted least squares with weights being the estimated standard deviations of residuals for each city (so there is cross section heteroscedasticity but no cross section dependence). We include a constant term to allow the conventional calculation of $R^2$ (which is 841). The coefficient estimates and standard errors are very similar for weighted least squares and panel based GLS, and the weighted least squares results also satisfy diagnostic tests. With Anderson’s small sample log likelihood correction, the restrictions in the final model are also accepted against both the initial 108 and the base 67 coefficient model.

There is an argument that there may be some endogeneity in the regressors especially in the wage gap variable. Shocks in net migration may feed back through the city labour market into shocks in the real wage. Thus the wage gap variables may be correlated with the net migration disturbances. We instrument the three wage gap variables by FDI and employment for the common group of cities and for cities 9 and 22, giving 6 instruments in all (the Sargan test for overidentification has a $p$ value of 0.265) and perform a Hausman-Wu test of the difference between the IV and the OLS estimates. It is not significant (the $p$ value is 0.42), and so we conclude that there are no significant feedback effects between net city migration and the city wage gap variable (Tab 6).
Table 7: GLS 27 coefficients

| GLS_27 coefficients |  
|---------------------|---------------------|
| NM(person)          |                      |
| Real urban expected urban wage gap(yuan) | 1.133*** (4.15) |
| Population gap(person) | -0.764*** (-10.03) |
| City1&22&23 Singel female gap(Person) | 1.207*** (5.13) |
| Urbanization level gap(urbanhukou) | 1.769*** (11.68) |
| City2 constant effect | 4.973*** (4.49) |
| City2 urbanization level gap(urbanhukou) | 5.516*** (4.16) |
| City3 constant effect | 0.174*** (7.62) |
| City6 constant effect | 0.102** (2.60) |
| City7 population gap(person) | 2.110*** (3.48) |
| City7 urbanization level gap(urbanhukou) | -2.122*** (-3.52) |
| City8 constant effect | 0.562*** (4.96) |
| City8 Capital stock gap(yuan) | 0.298** (2.76) |
| City9 constant effect | 0.374*** (5.67) |
| City9 Real urban expected urban wage gap(yuan) | 0.767*** (10.17) |
| City9 Capital stock gap(yuan) | -0.470*** (-6.99) |
| City11 constant effect | 0.0317** (3.22) |
| City13 Capital stock gap(yuan) | 0.161 (1.58) |
| City13 urbanization level gap(urbanhukou) | -0.351** (-3.22) |
| City14 urbanization level gap(urbanhukou) | 0.178*** (6.35) |
| City16 urbanization level gap(urbanhukou) | 0.315*** (7.74) |
| City18 constant effect | -0.146*** (-5.12) |
| City22 constant effect | 4.448*** (5.00) |
| City22 real urban expected urban wage gap(yuan) | -2.438*** (-7.62) |
| City22 population gap(person) | 8.206*** (4.70) |
| City22 Capital stock gap(yuan) | -1.022*** (-5.97) |
| City23 urbanization level gap(urbanhukou) | 14.94*** (7.68) |
| City23 Capital stock gap(yuan) | -0.440*** (-4.28) |
| Observations | 144 |

T-statistics are in parentheses

* p<0.05  ** p<0.01  *** p<0.001

Figure 38: GLS 27 coefficients Y vs predicted Y
In this reduced model all cities except for 9 and 22 have a common positive effect of expected wage differences. The expected wage gap does affect net migration into cities 9, 22 but to a smaller extent than in the other cities. Only a few cities have a responsiveness of net migration to the level of capital stock (cities 8, 9, 13, 22 and 23) and in these cities there are heterogeneous reactions to capital stock. The population gap is important in affecting net migration in all cities but only the effects in cities 7 and 22 are positive and heterogeneous whereas in the other 16 cities the response to the gap is negative although quantitatively small. All cities have net migration effects of their degree of urbanization, this is an equal effect in 12 cities but there are heterogeneous effects in 6 cities (cities 2, 7, 13, 14, 16 and 22). The effect of the gaps is thus common for most cities and most gaps.

The city specific constant terms in Table 5 and 7 reflect a relatively constant stream of net migration which is due to unobservable or non-measured city amenities (Davies et al., 2001). These factors are not determined by the operation of the gaps. These effects are important in half of the cities and in the majority of these cities there is inward migration which is not related to the gaps that we have identified.

Generally the gaps work in a way that is consistent with the theory\textsuperscript{18}: the population gap is a broad exception but its role generally is dominated by the merged city 22 which is very much larger than the other cities in terms of population. There are some other specific exceptions like the negative impact of the urbanization gap on net migration into city 7, indicating that, for that city, the hukou migration cost element outweighs the benefits of urbanization. Most of the heterogeneous gap effects can be explained in terms of special city characteristics. City 2 has been one the fastest growing cities in terms of capital stock and net migration. It is a high urban wage, densely populated and highly urbanized location. City 7 is a northern mountainous city with low urban wage and urbanization, high population but low population density and low capital stock. It shows high emigration. City 8 has a relatively high capital stock and low population and its textile industry base does not yield very high expected urban income, nevertheless it attracts immigration. City 9 is a coastal city and is the main Guangdong seafood producer with other industry concentrated on shipping construction. It is a low population and low population density city, with a low expected urban income and low capital stock, but despite this it has mean positive net migration. Foshan (13) is one of the industrial tigers with high expected urban income, capital stock and population and a relatively high degree of urbanization. It attracts positive net migration but is neither the leading nor second city in terms of the gap rankings. Cities 14 and 16 are low expected urban wage, relatively rural cities with low capital stock and average or low population density. Their mean net migration is close to zero. The merged city 22 stands out as having the greatest amount of specific heterogeneity in the migration response to gaps. As stated above

\textsuperscript{18}Since we have scaled the regressors to have zero mean and unit variance across the whole sample, the estimated coefficients are largely independent of the units in which we measure variables. So we do not compute distinct elasticities.
it dominates the other cities in population size but is relatively non-urbanised although it has a high population density. It also has low capital stock and at best average expected urban income. Its mean net migration is close to zero. Finally city 23 is a similar administratively merged city, sharing many of the characteristics of city 22.

3.5 Conclusions

Individuals are in heterogeneous circumstances and any one individual is affected by many different utility relevant variables. If individuals can locate in different possible places which also have heterogeneous characteristics, then we would expect movement of individuals between locations. We develop such a multi-motive and multi-location theory to determine the aggregate net migration flows between locations. We add the assumption that all individuals have identical preferences defined over multiple location specific characteristics and, at a general level, derive the result that there will be a tendency to agglomeration. All individuals will agree on a ranking of locations from best to worst. Those individuals with bad experiences in their current location will gain the most by moving to the location which is universally judged the best. Individuals who have current utility above the average for their present location may prefer to remain in situ especially if the migration cost is substantial. So less attractive locations will have emigration especially of the lowest utility inhabitants while the best location will have inward movement. Individuals who start off in the best location, but whose individual experience in that location is much below the location average may find it advantageous to move into the second most attractive location. So on balance the second best location may have net immigration or net emigration. We would expect agglomeration into the top two cities to occur. This matches up with some of the settlement patterns predicted in economic geography type models.

There are some theoretical innovations in migration modelling. Our approach allows for multivariate determinants and multi-location choices of net migration flows. People move to places where the chance of an improvement of their current circumstances in some dimension is highest. We confirm the basic Harris-Todaro insight that expected labour income differences are important but also confirm Krugman’s view that each location has a variety of push and pull factors determining migration.

We then use this framework to study the net migration flows between 18 different regions of the Guangdong province in China. Guangdong is particularly suitable for this purpose since it has experienced very rapid growth and industrialization in conjunction with high levels of inward immigration from the other Chinese provinces, and even higher levels of intra-province migration. We divide Guangdong into 18 city areas which have varying degrees of urbanization and use panel data on these 18 cities for 1990-1999 to econometrically investigate net migration flows between the cities allowing for cross section heteroscedasticity. We find that net migration into the majority of cities can be well explained by a common set of parameters. There is some limited heterogeneity between cities.
in how net migration responds to the differentials, out of a potential total of 90 city-differential heterogeneities we find that we need just 15 specific coefficients. The remaining heterogeneities are in the impacts of capital stock and the degree of urbanization. Nearly half of the cities share a common mean amount of net migration which is unrelated to the four differentials we identified. No cross section dependence and serial correlation are detected in the final model. In terms of goodness of fit and tracking the data city by city, our model performs well and there is no evidence of model misspecification.

In a locational equilibrium, the net benefits of moving between cities should be equalized. In fact inequality between cities in some of the relevant factors has increased not fallen over our sample. The coefficient of variation of urban/rural income and capital stock per capita suggest that inequalities between cities are increasing over time. Taken together, the rising inequality in some migration inducing factors may imply that a full locational equilibrium has not yet been achieved.

It is well known that Chinese labour migration is substantial and exhibits different types of flows. It is widely argued to be a very important component in rapid Chinese growth and development, thus its policy importance is clear. Although the data sources are much more abundant than 20 years ago, there is still a paucity of degrees of freedom and coverage of some of the relevant factors. This forces some imperfection in our modelling strategy, but, given this, the results here are robust to a range of specification tests.

### 3.6 Some further thoughts

Due to micro data limitation, we made some strong assumptions in the theoretical framework such as the same agreement on city ranking among individuals. Now we have some rich microdata sets at hand, we could relax those assumptions and apply it to the microdata sets. The framework we use is static and one period. We abstract from temporary migration, planned reverse migration and commuting/guest working. We also work with an individual as the decision maker which allows us to avoid specifying family decision processes and to derive aggregate net migration equations in a multi-motivation, multi-destination setting. A partial justification is that much aggregate migration data is at the household not individual level, and abstracting from intra-family decision rules yields empirically testable equations. However clearly a next step would be a multi-period and family based model. In a two adult household with one dependent child, the migration choice will be made based when the gain of reallocation in destination exceeds the overall household utility in origin.

We could allow the wage rate to adjust in each location according to labour demand and supply and add non-random matching frictions (or searching frictions) for migrants. From the supply side, migrants are still utility maximizers who are seeking for the highest net return of relocation based on the gain they could get in the host
places, the benefit they need to forego in the origin and the matching friction. The matching friction can be either
due to characteristics (e.g. low skill to search for a job), or asymmetric information, or market discrimination
against migrants, or high level of moving costs, or big entry barriers (e.g. hukou in China). All these exogenous
factors cause non-random matching frictions. From the demand side, firms are profit maximizers who employ both
migrants and natives. If there is a high level of discrimination against migrants in the local market, then firms will
employ more natives than migrants. Otherwise, migrants and natives face equal opportunities for the same job.
The current wage rate is endogenously determined by the past supply and past demand. The market equilibrium
is achieved when the past supply equals to past demand.
4 Intergenerational transfer and education investment in an altruistic-OLG frameworks

4.1 Introduction

This chapter tries to answer question 4 of Chapter 1 and builds a self-sustained system to look at the effects of education on economic development and inequality in a very simple economic environment. We assume individuals have reciprocal altruistic preferences. By having altruistic preferences, we want to look at how the coordination works between individuals. Can it promote economic development and tackle inequality problems? The relatively wealthy individuals can help increase the welfare levels of the poor ones by giving some wealth to the poor. Typically the young are poor, they are just starting to build their wealth but are very likely to face liquidity constraints. Without some redistribution those individuals would be stuck at the initial point. Redistribution can prevent this but the process normally needs help from either a government or a central planner, who can implement the allocation for example through a lump sum taxation/subsidy system. This requires the government to know who is liquidity constrained and poor. But we want to simplify the economic environment, saving much of this information cost. So we want to develop a model in which individuals are self-motivated to send money without being forced to by a government tax system. Obviously, an OLG (overlapping generation model) with altruistic preferences but without government intervention is a good framework. At each period, the relatively wealthy generation transfer some wealth to the relatively poor generation giving them the chance to move up. Then the next question is the use of the wealth. Of course, the relatively poor generation can just leave the wealth under the bed and use it later or do some investment or just consume it. There are so many possible investments such as property investment and educational investment. In this chapter, we focus on the educational investment. As we know, there is a positive link between human capital and lifetime wealth (Becker, 1974).

This generates our interest in developing analytic frameworks using household decision theory to understand intergenerational transfers and education investment in an altruistic-OLG setting. Besides the motivations mentioned above, we are also stimulated by the following other aspects: (1) the social security idea has been well developed but the family altruism idea still leaves an "open box". Instead of implementing regulation and law, can family altruism easily generate binding social contracts? In a society, if each household lives in a caring/love environment, then can a social optimum be achieved within a small unit such as a family? This leaves us a research question; (2) most of the current research focuses on one way altruism and two way altruism has not been so widely studied. For instance, reciprocal altruism is one interesting way to justify the reason why individuals are altruistic towards others. Because the altruistic individuals know that they will receive the return benefits next time. The OLG is a
natural home to study this as we need a dynamic setting to allow individuals to care about others in one period but also be cared by others in the next time period; (3) the OLG could be expanded to consider intertemporal education investment to discuss the effect of the altruistic behavior on education investment. In particular, can family altruism be the vehicle to make the private Nash Equilibrium socially optimal? (4) empirical evidence in developing countries requires a theoretical framework to understand the dynamic link between intergenerational transfers and education investment.

An increasing number of Chinese students have obtained higher education from elite universities in the US\textsuperscript{49} and Britain\textsuperscript{50} in the last 10 years. Those overseas students are considered to be the most fortunate generation named as "sea-turtles" returning to China with a strong advantage in the labor market. Most of those students are financially supported by their family in which parents are relatively rich and highly educated. Education has always been extremely important to the Chinese. Within the one child policy, parents pay lots of attention to the education of their kids and are willing to invest anything they can in the kids’ education. Meanwhile, those "sea-turtles" graduate from elite universities with BA or MA degrees. These elite degrees allow them to either get jobs abroad or to be employed in China with high pay. The high pay lets them financially support the elderly in the form of free gifts as well as help the new young generation with human capital investment, and the amount of transfers depend on the degree of the two way altruism. This causal link between education investment and income and the relationship between income and free gifts are worth studying. For instance, the causal links run from a high education investment at $t - 1$ to a high income level at $t$. The high income level at $t$ can finance transfers to the next generation to fund their education. And further this education investment at $t$ further increases income level at $t + 1$. Meanwhile, the high income level at $t$ can fund the free gift to the previous generation. This dynamic effects can be presented as below,

\begin{align}
... \rightarrow & \text{Education investment}_{t-1} \rightarrow \frac{\text{Income}_{t}}{(2):\text{Free gift}_{t}} \rightarrow \text{Education investment}_{t} \rightarrow \text{Income}_{t+1}...
\end{align}

The willingness to do investment on kids’ education has been demonstrated by the fact that there are an increasing number of Chinese students studying abroad. But the incentives (which can be altruistic) to do investment on kids’ education have rarely been analyzed. The altruistic incentives can be derived from either warm glow (e.g. Andreoni, 1990, 2006), or social expectation (e.g. Olson, 1965), or bounded rationality/myopic love (Becker, 1974)

\textsuperscript{49}Over the past five years, most of the growth in international enrollment has come from China, while the numbers from other top-10 sending countries have remain virtually flat*. The number of mainland Chinese students enrolled at colleges across the United States increased from around 800,000 to 1900,000 between 2007 to 2011 (Source: Institute of International Education http://chronicle.com/article/China-Continues-to-Drive/135700/).

\textsuperscript{50}In UK, China (PRC) and India rank no 1 and no 2 among top 10 non-EU sending countries for international students. The number of Chinese students (78,715) is twice that of Indian students (29,900) in 2011-12. (source:http://www.ucas.org.uk/about/statistics_he.php)
or pure love/altruism (Becker, 1974; Konow, 2006) or a mixture of them. All these incentives\footnote{The differences between these three scenarios lies in the explanation of why altruistic agent sends free gift and does HC investment. (i) In the censored altruism case, the reason is down to some social norm or custom or self-reputation which compels the M to do so. (ii) In the semi-altruism case, the reason is explained by the bounded rationality or myopic love in which the preference of the donor depends only on the current observable utility (or the current consumption) of recipients. The donor cannot foresee the recipients future state. (iii) In the pure altruism case, the reason is due to pure love as the M cares about the lifetime utilities of others.} will be studied in detail.

The analysis of the intergenerational transfers and education investment depends on the nature of the household model, on the incentives for transfers and on the behavior of the system. For the nature of the household model, we treat a household as a unit containing three generations (young, middle aged and old) living in the same family with independent and dependent decision makers and with working and non-working generations. The altruistic generations are those who have altruistic preferences and make intergenerational transfers. The intergenerational transfers are normally in the form of gifts and education investments and the amount of the transfers vary with the degree of altruism. For the behavior of the system over time, we apply OLG in a decentralised economy in which three generations live in the same household with continuous new born young generations, each generation lives three periods and there are economic interactions between generations because of transfers.

In terms of the relevant theoretical frameworks, there are a few worth mentioning in a general way. First, Docquier et al. (2006) study the dynamics of bequeathed wealth in the unitary model of household’s migration decision in the fourth part of their paper. They discuss the link between intergenerational transfers and migration not the relationship between intergenerational transfers and education investment. The advantages and disadvantages of the unitary model are well displayed in their paper. The general story is that in each time period, there is a one period lived household in an open rural economy with access to labor market. The household head decides that some of the family members are sent to the labor market and have to pay for the moving cost and some of them are kept for agricultural production. Those sent to the labor market are considered as migrants and they contribute all their income to the rural household. And those migrating face a moving cost from the rural to the labor market. Paying for this cost and hence migration is constrained by the wealth inherited from the previous generation. The more wealth inherited from the previous generation, the more family members can be sent to the labor market to get high pay with a wage $w^*$ and the more family wealth can be accumulated. Also the more wealth can be left for the next generation. Thus, in the dynamic process of bequeathed wealth, they get $b_{i+1} = f(w^*; b_i, \alpha, c, \delta)$, where $b_i$ means the bequeathed wealth at time $i$ and $\delta$ is the intergenerational altruistic parameter; $\alpha$ is agricultural productivity of the household; $c$ is the collective moving cost faced by the household; $f$ is the level of wealth including the market wage, inherited wealth/bequest. Depending on the values of those parameters, the steady states are also going to vary. Actually, in their paper, they split the rural household into two groups: low productivity (LP-$\alpha$) and high
productivity (HP: $\alpha$), which face different bequeathed wealth ($b_i$ and $\overline{b}_i$). These two productivity levels originate from the quantity and quality of the inherited family land and cannot be changed by accumulated family wealth. These two productivity levels act as the reservation wage for migrants and determine different migration incentives. The HP households face a high reservation wage so that the migrants in those HP households require a higher market wage ($w^* > \alpha > c$) while the LP households face a low reservation wage so that the migrants in those LP households choose to move as soon as the market wage is high enough ($\alpha < w^*$). But both types of household are assumed to face the same market wage ($w^*$) and individual moving cost ($c$). In particular, the HP households are less constrained by moving cost $c$ but less likely to move unless $w^*$ is high enough. Even though the HP households have a bigger amount of inherited wealth so that they are less liquidity constrained, they face higher reservation wage requiring higher market wage. In the end, they find different steady states for those two types. The details of the two types will not be addressed further in this chapter but the logic of the dynamic process of bequeathed wealth for those two types is similar to the general dynamics: $b_{t+1} = \delta + f(w^*, b_t, \alpha, c)$.

The accumulated wealth across generations seems to be explained well in the unitary setting. But there are some flaws in this model such as too simplistic assumption of constant marginal productivity and homogenous moving cost for LP and HP households. However, there are a few main issues that are addressed here:

- From a theoretical point of view, the major limit is that in unitary model family members’ resources are pooled together; however, no general framework has been developed in order to explain how consensus on pooling and on intra-family allocation is achieved. As soon as there is an intra-family transfer or allocation, the unitary model won’t be able to explain the allocation process. So we need a better model (e.g. non-cooperative or collective household model) to explain the allocation process. This is one reason why we do not use the unitary model in this chapter.

- In their framework, every household consists of the same given number of two-period-lived agents. Parents have complete decision power over children, some children are sent as migrants who must send all their income back. Other children remain working in the rural area and again their income is pooled into the family. Current parents die and the children establish their own family, the cycle repeats. Is it really self-sustaining? why should current children return all income to the family? We use a broader base of altruism to motivate transfers between the two generations.

Secondly, Moav (2005) studies the evolution of the intergenerational education in the unitary model of a OLG environment. In each period, a generation of young children is born and has a single parent. Individuals live two
periods: in childhood they acquire human capital; in adulthood they allocate one unit of time between child rearing and work. The young generation is not an independent decision maker and is set to put all his childhood time on education. The outcome of education investment at $t$ is realized in adulthood at $t + 1$. The single parent decides the amount of education investment on this young generation, meaning that the young generation’s human capital is an increasing function of the single parent’s investment ($h_{t+1,t} = h(e_{t,t-1}^P)$, where $h_{t+1,t}$ is the human capital level of the young generation at time $t + 1$ who is born at $t$ and $e_{t,t-1}^P$ is the education investment invested at time $t$ by the single parent born at $t - 1$. The available wealth ($w * h(e_{t-1,t-2}^P)$) of the parents depends on the constant market wage rate $w$ and their human capital ($h(e_{t-1,t-2}^P)$) arising from education investment they received, which was funded by their parents ($e_{t-1,t-2}^P$). At time $t$, the maximization problem for the single parent is to choose the number of children they want to have, the education investment they want to undertake and their own consumption subject to their income, that is

$$\max_{n_t, e_{t,t-1}^P, C_{t,t-1}^P} w_{t,t-1} = \max_{n_t, e_{t,t-1}^P} \{ (1 - \beta) \log(C_{t,t-1}^P) + \beta[\log n_t + \theta \log wh(e_{t,t-1}^P)] \}$$

s.t. 

$$w * h(e_{t-1,t-2}^P) \geq n_t [\tau H_{t,t} + e_{t,t-1}^P] + C_{t,t-1}^P$$

where $C_{t,t-1}^P$ is the consumption at time $t$ of the single parent born at $t - 1$; $n_t$ is the number kids in this household at time $t$; $h(e_{t,t-1}^P)$ is the level of human capital of each child measured in efficiency units which is determined by the single parent’s education investment $e_{t,t-1}^P$; $w$ is the constant market wage rate per efficiency unit of human capital; $\beta$ captures the relative weight given to children (quality and quantity); $\theta$ is the relative weight given to children’s quality in the utility function; $\tau$ is the minimum time cost required for raising a child who is endowed with $H_{t,t}$ efficiency units of initial human capital; $e_{t,t-1}^P$ is the cost of education investment. In this unitary model, the benevolent parent derives utility from his own consumption and the potential benefit from raising kids. This benevolent parent has to face the cost of children quantity ($n_t \tau H_{t,t}$), as well as he has to face the cost of children quality ($n_t e_{t,t-1}^P$). Overall, the exogenous parental income is spent either on caring and educating children or on their own consumption. By solving this maximization problem, Moav gets $e_{t,t-1}^P = \phi(e_{t-1,t-2}^P, \tau, \beta, \theta)$ (Fig 39).

In order to simplify the notation, we write the dynamic equation as $e_t = \phi(e_{t-1}, \tau, \beta, \theta)$, the current period of education investment ($e_t$) is determined by the previous education investment ($e_{t-1}$), the minimum time cost $\tau$, degrees of altruism ($\beta$ and $\theta$). By varying those parameters, the number of steady states is also going to change.

The evolution of the intergenerational education in Moav’s (2005) model is a insightful dynamics. But there are some issues addressed here:

- The setting of the altruism can be revisited. In Moav’s paper, the altruism is only one way from parents
to children. We can extend the one way altruism to reciprocal altruism in the sense that the children can reciprocate parents’ help in the next time period. The reciprocal altruism mechanism can enrich the theoretical modelling and may lead us to discover other new properties of altruism.

- The single parent’s time allocation is not clearly discussed in the modelling part as the single parent needs to face time allocation budget constraints (between child care and working at the labor market). E.g. the single parent’s total time endowment is $T$, then he spends $L$ on his work and $T - L$ on child care. Then the budget constraint could be $w \cdot L_{t,t-1} \cdot h(e_{t-1,t-2}) \geq n_t \cdot (T - L_{t,t-1}) \cdot w (\tau h_{t,t}^C + e_{t,t-1}^P) + C_{t,t-1}^P$.

Overall, the introduction has discussed (1) the motivation of our research, (2) the nature of the household model, the incentives for transfers and the behavior of the system over time, (3) two relevant theoretical frameworks. Those imply (i) that there are theoretical limitations of the previous theoretical works (e.g. unitary model), alternative approaches should be proposed and analysed. (ii) The analysis of intergenerational transfers and education investment has some potential research value, as the analysis reflects the real background story (Chinese "sea turtle") and also has policy implications. (iii) The altruistic behavior has been discussed in previous literature but whether or not the private NE can be social optimum for censored altruism has not widely been discussed. (iv) The one way altruism can be expanded to reciprocal altruism. (iv) The OLG seems to be a feasible environment to analyze individual decisions in a decentralized economy.

In the following parts, section 2 provides a literature review and section 3 presents our model in detail and discusses about the possible policy. Section 4 concludes. Section 5 proposes some future research.
4.2 Literature Review

4.2.1 Household modelling

Micro-economic theory often considers the household as a single decision unit, with well defined preferences maximized over a total household budget constraint. And the consumption of the household is not affected by the income distribution within the family. This setting has been increasingly challenged in the last decades, when a growing number of researchers (Browning et al, 2011; Browning and Chiappori, 1998; Lundberg and Pollak, 1993) have shown the theoretical limitations as well the shortcomings on empirical evidence. So alternative household modelling approaches have been proposed as individuals have their own preferences and budget constraints. According to this view, the household decision process can be defined as either the outcome of a non-cooperative game or a cooperative game (collective approach). These frameworks develop tools for dealing with the allocation problem within a household.

The present section is organized as follows: the unitary model is analyzed; the non-cooperative and cooperative approaches are reviewed.

The unitary model of household behavior

The unitary model can either be based on a household utility function, defined over the consumption of the different family members, or through a family welfare function as a combination of individuals’s utilities, such as the Bergson Samuelson welfare function depending on the family members’ utility functions. In both cases, decisions are taken to maximize the household utility/welfare function subject to the household budget constraint. We will examine each of them in turn.

Household utility

In the unitary model of a household utility, it is assumed that the household behaves as if it maximizes a unique, price independent utility function subject to a family budget constraint. Consider an N-member family with members indexed $i$. Member $i \in N$ consumes a homogenous private good $c_i$ at price $p$. Let $y^i$ denotes member i’s income. This household chooses consumptions as follows,

$$\max_{c_1,...,c_N} \left[ u(c_1,...,c_N)|y^1 + ... + y^N \geq \sum_{i=1}^{N} pc_i \right]$$

where $c_i$ ($i = 1...N$) is the consumption of each member and the utility function $u(\cdot)$ is a strictly quasi-concave and twice differentiable in its arguments.

The Bergson Samuelson welfare function (BSWF)

The Bergson Samuelson welfare function (BSWF) approach can be traced back to Samuelson (1956). He notes
the "impossibility of group or community preference curves" problem of family preference, meaning that it is not very realistic that a number of individuals living in the same group or community or family has the same tastes unless the individuals have the same marginal rate of substitution at all points. In economics, we normally define the marginal rate of substitution as the slope of an indifference curve. And the slope of the indifference curves is a device to represent local preferences between different goods. In a two consumers and two goods economy, if those two consumers always have different MRS towards those two goods, then they always face different exchange rate for good X and Y. So that they have different ordinal preferences.

However, since blood is thicker than water, the preferences of the different members are interrelated by a BSWF which takes into account the preferences of each of the members. The family acts as if it were maximizing their joint welfare function. Traditionally this is defined by a function \( W : \mathbb{R}^N \rightarrow \mathbb{R} \) that maps utility of each individual, \( u^i \), into a composite welfare function, \([u^1(q^1_1, \ldots q^1_m), \ldots u^N(q^N_1, \ldots q^N_m)] \rightarrow W\). This either requires cardinal and comparable utilities of individuals, or that \( W \) is only defined for some particular monotonic transformation of each individuals utility.

Concretely, assume that a given household consists of \( N \) individuals with a BSWF, its decisions are taken to

\[
\max_{q^1_1, \ldots q^m_m} W[u^1(q^1_1, \ldots q^1_m), \ldots u^N(q^N_1, \ldots q^N_m)]
\]

subject to

\[
\sum_{h=1}^{m} \sum_{i=1}^{N} p_h q^i_h = \sum_{i=1}^{N} y^i
\]

where \( p_h \) is the price for private good \( h \) (\( h = 1 \ldots m \)); and \( q^i_h \) is person \( i \)'s private consumption for good \( h \); \( y^i \) is person \( i \)'s individual income.

There are different ways of interpreting this household decision procedure. One interpretation is the quasi-independence as if every one is regulated by an invisible hand to have a mutual agreement on maximization of the family welfare function and follow an agreed rule for the income distribution. In this case, the maximum family welfare can be achieved through two stages:

Stage 1: In order to maximise family welfare function \( W \), each family member agrees any arbitrary income distribution rule that family wealth is redistributed and each obtains a new income \( M^i \) (\( \sum_{i=1}^{N} M^i = \sum_{i=1}^{N} y^i \)). Then, given the new income, each maximises his own utility subject to the budget constraint.

\[
\max_{q^1_h, \ldots q^m_h} [u^h(q^h_1, \ldots q^h_m)p_h q^i_h = M^i]
\]
achieving indirect utility

\[ v^h(p_h, M^i) \]

Stage 2: The whole household maximises \( W \) subject to household budget constraint to choose the optimal income distribution,

\[
\max_{M^1 \ldots M^h \ldots M^n} \left[ W(v^1(p_1, M^1) \ldots v^h(p_h, M^h) \ldots v^n(p_n, M^n)) \right] \sum_{i=1}^N M^i = \sum_{i=1}^N y^i
\]

Another view might see \( u^i \)’s as the agents individualistic preferences, and that \( W \) is the utility function of an altruistic head of household which is similar to Becker’s altruistic model (the "Rotten Kid Theorem"). The whole household behaves as if it were governed by a single, utility maximizing decision maker who represents the trade-offs between different individual preferences.

It is worth noting that this BSWF approach does conform to the requirement that each member’s separate tastes count.

**Becker’s altruistic model**

Becker briefly discusses the altruistic household model in his 1974 paper: one family member, the household head, is benevolent toward all other family members (e.g. selfish kids or rotten kids) and is rich enough so that he can give some money to each family member to maximize the family welfare, which coincides with his own altruistic preferences. There is a single consumption good \( X \) and each member consumes \( X^i \) amount. All family members are selfish except the household head who is altruistic. This altruistic household head’s utility depends on the consumption of each member. Therefore, the household head’s utility can be written as,

\[
U(X^1, \ldots X^N)
\]

The budget constraint of the total family consumption is

\[
\sum_{i} p_i X^i = \sum_{i} y^i
\]

where \( y^i \) is the income of member \( i \) before any intra-family transfers happen; \( p \) is the price of this single consumption good \( X \).

Then if the household head makes intra-family transfers to all other members, then after the transfers are made, the distribution of consumption in the family will be the one that maximises the head’s utility subject to the family budget constraint. The whole house household behaves as if it were governed by a single, utility maximising decision maker. The family consumptions will be the vector \( (X^1, \ldots X^1 \ldots X^n) \) and each \( X^i \) is a function of family income, \( X^i(p, \sum_{i} y^i) \). Given the reasonable assumption that the \( X^i \)’s are all normal goods, then \( X^i \) is a monotonic increasing
function of total family income, $\frac{\partial X^i}{\partial \sum_{j \neq i} Y_j} > 0$. Therefore, the rotten kids do not have any incentives to deviate or to behave badly. Even though they are selfish, they know that the only way to increase the $X^i$ after family transfers is increasing total family income. By knowing this, it is in the self-interest of all selfish members to maximise the total income of the household and to increase their own private incomes. However, this theorem holds under certain restrictive assumptions: the head is altruistic, rich enough, each member only has one consumption good and each lives only one period, then the selfish kids will act efficiently from the family viewpoint and act harmoniously in the family interest. However, it is worthwhile to point out its limitations,

- The case of lazy rotten kids (Bergstrom, 1989). Bergstrom (1989) shows that there is an incentive problem if the selfish kids' utility depends on two arguments (consumption good and working effort). He presents one household with two children, each child’s utility is $U_i = X_i(1 - Y_i)$, where $X_i$ is consumption good and $Y_i$ is $i$’s working effort. The head has income $I_o$ but does not work or consume anything. Then the altruistic head maximises the utility $U_o = U_1^{1/2} + U_2^{1/2}$ subject to family income constraints $X_1 + X_2 = I_o + w(Y_1 + Y_2)$. Through maximization, he gets $X_1/X_2 = (1 - Y_1)^2/(1 - Y_2)^2$, where one child’s consumption is a decreasing function of his own working effort and of the other child’s working effort. Therefore, in the example of Bergstrom’s paper there is an incentive problem for working hard. Thus, if the kids utility depends not only on the single commodity $X$ and but also another argument such as working effort (the selfish kids have two arguments in their utility function/preference), then the lazy kids will mainly rely on family transfers but have no motivation to work hard.

- Samaritan’s dilemma (Lindbeck and Weibull, 1988; Bruce and Waldman, 1990). They point out that this theorem does not hold in a multi-period model. In a two period framework with savings, if the parent makes transfers to the children in the second period, then the children will save too little (Samaritan dilemma type inefficiency) and consume a lot in the first period. The kids will mainly rely on family transfers in the second period. On the other hand, if transfers take place in the first period, the children will not be motivated to maximize family income in the second period (Rotten kid type inefficiency).

**Remark on the unitary model**

It is clear from the previous subsection that the unitary models are not satisfactory on theoretical grounds. Samuelson notes the "impossibility of group or community preference curves" representing family preferences, in the sense that a household cannot be modelled in the unitary way if its family members have different preferences. Becker’s approach, on the other hand, mainly considers the workings of the household as an economic institution,
in matters such as bequest and human capital formation and so on. But he simply assumes that in a single commodity economy there is an altruistic household head/dictator with preferences over the distribution of the single good between family members. But this simplistic assumption has not solved Samuelson’s social indifference curves problem. Becker mainly considers the rotten kids problem from a positive economic viewpoint. He has not explained why there is an altruistic head and why do rotten kids sign up to a constitution giving the dictator sole decision power? In addition, whether the dictators benevolent preferences (family preferences) are compatible with the private preference of each family member is not explained neither. If the social preferences cannot stand for the private preference, then Samuelson’s social indifference curves problem is not solved neither.

These concerns have led to the introduction of several models, built on the acknowledgment that the household consists of individuals exhibiting different preferences. These models address the issue of how those different preferences may be reconciled within the household. The literature has developed both cooperative and non-cooperative solutions to the problem giving rise to two different classes of models: the non-cooperative and the cooperative one respectively. We are going to review those two models in turn.

**Non-cooperative approach**  The basic assumption is that there is no binding commitment or agreement between individuals and each chooses his action independently.

A few previous papers apply non-cooperative game theory in a family setting. Browning et al (2011) reviews the non-cooperative model with and without public goods and interactions. Also, non-cooperative family models with public goods and family transfers can be found in Bergstrom et al. (1986). In the following part, we are going to review the works of Browning et al (2011).

**First we review the non-cooperative approach with no public goods discussed by Browning et al (2011).**

Individuals are assumed to be egoistic and all commodities are privately consumed. Each individual (person a and b) has his/her own preference and private budget constraint. The non-cooperative solution boils down to the following two programs:

\[
\max_{q^a} u^a(q^a) \text{ subject to } p'q^a = Y^a \tag{17}
\]

\[
\max_{q^b} u^b(q^b) \text{ subject to } p'q^b = Y^b \tag{18}
\]

where the consumption \((q^s, s = a, b)\) of individual \(s\) is simply this individual’s demands at prices \(p\) and income \(Y^s\). Denote the demand functions for \(s\) by \(\xi^s(p, Y^s)\). Given the aggregate resource within the household is \(Y^a + Y^b\), the allocation \((\xi^a(p, Y^a), \xi^b(p, Y^b))\) is Pareto efficient: if there is a social planner, there is no chance for the central
planner to move some resources from \( a \) to \( b \) to make \( b \) better off without reducing \( a \)'s welfare. The household demand function is

\[
\xi(p, Y^a, Y^b) = \xi^a(p, Y^a) + \xi^b(p, Y^b)
\] (19)

This household demand function does not satisfy income pooling\(^{52} \) or the Slutsky conditions\(^{53} \) and depends on the income distribution.

There are no ideal compensating aggregate income changes unless the distribution is determined by aggregate income. We can of course construct income distribution changes consistent with a given aggregate income change which will yield the necessary individual income compensations to keep individual utility constant. Then, since individual compensated demands satisfy Slutsky symmetry, the aggregate demand (with this associated income distribution change) will also satisfy Slutsky symmetry. For example with two individuals \( a, b \) and utilities \( u_a(q^a), u_b(q^b) \) the Slutsky equation for \( a \) is

\[
\frac{\partial q_i^a}{\partial p_j} \bigg|_{u^a \text{ constant}} = \frac{\partial q_i^a}{\partial p_j} + q_j^a \frac{\partial q_i^a}{\partial y_{ja}} = \frac{\partial q_i^b}{\partial p_j} + q_j^b \frac{\partial q_i^b}{\partial y_{ja}} = \frac{\partial q_i^a}{\partial p_j} \bigg|_{u^a \text{ constant}}
\] (20)

which follows from a compensating income change \( dy^a = q_j^a dp_j \) and similarly for \( b \). The aggregate demand for good \( i \) is \( Q_i(p, y^a, y^b) = q_i^a + q_i^b \) If the \( j^{th} \) price changes and we change aggregate income by \( dY = q_j^a dp_j + q_j^b dp_j \) and change the individual incomes by right amounts (so that \( a \) gets a change \( q_j^a dp_j \) then the aggregate quantity change

\[
\frac{\partial Q_i}{\partial Y} \bigg|_{u^a \text{ constant},u^b \text{ constant}}
\]

is the sum of the individual compensated changes and so must be symmetric in \( i \) and \( j \).

Besides, there is a special case in which income pooling and the Slutsky conditions\(^{54} \) hold for the non-cooperative approach, that is, the Engel curves are linear with a common slope

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\(^{52}\) The "income pooling" idea formally originates with Becker (1991) that sharing is independent of who actually bring the income into household. Or equivalently, the distribution of income within the household has no impact on household outcomes such as labour supply, savings and demand patterns.

\(^{53}\) Based on Slutsky (Barten, 1967), there is a differentiable utility function \((u = \sum_{i} u(q_i), i = 1, \ldots, n)\) and a budget constraint \((\sum_i p_i q_i = m)\), where \( p_i \) is the price of the commodity \( i \), \( q_i \) is the quantity purchased of the commodity \( i \) and \( m \) is total expenditure or income. The following four equations together are discussed in Barten’s paper. And Slutsky conditions mainly refer to properties (3) and (4).

\[
\begin{align*}
(1) & : \quad \sum_i p_i \frac{\partial q_i}{\partial m} = 1; \\
(2) & : \quad \sum_j \frac{\partial q_j}{\partial p_j} = -q_j; \\
(3) & : \quad \frac{\partial q_i}{\partial p_j} \bigg|_{\Pi} = \frac{\partial q_j}{\partial p_i} \bigg|_{\Pi}; \\
(4) & : \quad \frac{\partial q_i}{\partial p_i} - \frac{\partial k_i(p_i, \Pi)}{\partial p_i} = q_i \frac{\partial q_i}{\partial m} < 0
\end{align*}
\]

where properties (1) and (2) come from budget constraints; equation (1) implies that an increase in total expenditure is completely allocated to all commodities in the budget (Engel aggregation); equation (2) ensures that an change in one price leaves the amounts of commodities purchased unchanged (Cournot aggregation: price weighted sum of responses to a change in \( p_j \) equals to the amount of the \( j^{th} \) good consumed); equation (3) is the symmetry property, which means that the quantity changes in commodity \( i \) and \( j \) are the same (given that the utility is kept constant); based on "law of demand", equation (4) shows the negative link between price and demand for the same good \( i \) and the good is not giffen good \((\frac{\partial q_i}{\partial p_i} < 0)\) and is normal good \((\frac{\partial q_i}{\partial p_i} \geq 0)\). \( k_i(p_i, \Pi) \) is Hicksian demand.

\(^{54}\) \( c^{i,h} = \phi^{i,h}(p) + \varphi^i(p)y^h \), where \( i \) stands for good \( i; h \) means individual consumer.

Then we know individual demand function satisfies Slutsky conditions:

\[
\begin{align*}
(\frac{\partial \phi^{i,h}(p)}{\partial p_i} + y^h \frac{\partial \phi^i(p)}{\partial p_i}) + \varphi^i(p)[\phi^{i,h}(p) + \varphi^i(p)y^h] \\
= (\frac{\partial \phi^{i,h}(p)}{\partial p_i} + y^h \frac{\partial \phi^i(p)}{\partial p_i}) + \varphi^i(p)[\phi^{i,h}(p) + \varphi^i(p)y^h].
\end{align*}
\]

The total demand for good \( i \) is \( C^i = \sum_h c^{i,h} = \sum_h \phi^{i,h}(p) + \varphi^i(p) \sum_h y^h = \sum_h \phi^{i,h}(p) + \varphi^i(p)Y. \)

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\[ c_i = \phi^i(p) + \varphi(p)y^i \]  

This is equivalent to each consumer having quasi-homothetic preferences with a common marginal effect in income but a different constant term. Each consumer has the same slope \( \varphi(p) \) and one private constant term. The private constant term \( \phi^i(p) \) is not affected by income and reflects the heterogeneity of preference.

So the household demand for this private good is given by:

\[
C = \sum_i \phi^i(p) + \sum_i \varphi(p)y^i
\]

\[
= \sum_i \phi^i(p) + \varphi(p)(y^1 + \ldots + y^N)
\]

Thus, the aggregate family demand is independent of the income distribution.

And each member equally responds to the income change \( (Y = \sum y^i) \)

\[
\varphi(p) = \frac{\partial c_i}{\partial y^i} = \frac{\partial C}{\partial Y} \frac{\partial Y}{\partial y^i}
\]

This is a very special case in the sense that the household demands do not depend on the distribution of income but the distribution of the goods within the same family depends on the distribution of income.

Secondly, we review the non-cooperative approach with public goods also discussed by Browning et al (2011).

When an interaction between members is introduced into the non-cooperative approach, there are two cases that could be considered as follows.

(1) Consider a two person family, each contributes to the single public good \( (Q = Q^a + Q^b) \) at price \( P \) but consumes the private good \( (q) \) at price \( p \) separately.

\[
\max_{Q^a, q^a} \{u^a(Q^a + Q^b, q^a)|PQ^a + pq^a = Y^a\}
\]

\[
\max_{Q^b, q^b} \{u^b(Q^a + Q^b, q^b)|PQ^b + pq^b = Y^b\}
\]

Then \( \frac{\partial C^i}{\partial y^j} + C_j \frac{\partial C^i}{\partial Y} = \left( \frac{\partial \sum_h \phi^{i,h}(p) + \varphi(p) Y}{\partial y^j} + Y \frac{\partial \phi^i(p)}{\partial y^j} \right) \)

\[
+ \left( \sum_h \phi^{i,h}(p) + \varphi(p) \sum_h y^h \right) \varphi^i(p) \frac{\partial (\phi^{i,h}(p))}{\partial y^j}.
\]

\[
\frac{\partial C^j}{\partial y^j} + C_i \frac{\partial C^j}{\partial Y} = \left( \frac{\partial \sum_h \phi^{j,h}(p) + \varphi(p) Y}{\partial y^j} + Y \frac{\partial \phi^j(p)}{\partial y^j} \right) \]

\[
+ \left( \sum_h \phi^{j,h}(p) + \varphi(p) \sum_h y^h \right) \varphi^j(p) \frac{\partial (\phi^{j,h}(p))}{\partial y^j}.
\]

It is obvious that \( \frac{\partial C^i}{\partial y^j} + C_j \frac{\partial C^i}{\partial Y} = \frac{\partial C^j}{\partial y^j} + C_i \frac{\partial C^j}{\partial Y} \).

Therefore the aggregate demand satisfies the Slutsky conditions.
These two maximization problems give reaction curves

\[ Q^a = Q^a(P, p, Y^a, Q^b) \]  \hspace{1cm} (26)\n\[ Q^b = Q^b(P, p, Y^b, Q^a) \]
\[ q^a = q^a(P, p, Y^a, Q^b) \]
\[ q^b = q^b(P, p, Y^b, Q^a) \]

Through this we can get a Nash Equilibrium between \( a \) and \( b \) for private consumptions and household public consumption

\[ q^a_{NE} = \xi^a(P, p, Y^a, Y^b) \]  \hspace{1cm} (27)
\[ q^b_{NE} = \xi^b(P, p, Y^a, Y^b) \]
\[ Q_{NE} = \epsilon(P, p, Y^a, Y^b) \]

In general the income distribution will affect the NE solutions but of course the NE values will satisfy the household budget constraint, since the individual best responses satisfy the individual budget constraints, by summing the individual budget constraint

\[ P(\hat{Q}^a + \hat{Q}^b) + p(\hat{q}^a + \hat{q}^b) = Y^a + Y^b \]

where the \( \hat{\cdot} \) means the NE values. Ex-post in the solution, then the income pooling property hold for both the private and the public consumption.

(2) We consider a case when only one person contributes. Let’s assume this person is \( a \),

\[ \max_{Q^a, q^a} \{ u^a(Q, q^a) | PQ + pq^a = Y^a \} \]  \hspace{1cm} (28)
\[ \max_{q^b} \{ u^b(Q, q^b) | pq^b = Y^b \} \]

we get

\[ q^a = \xi^a(P, p, Y^a) \]  \hspace{1cm} (29)
\[ Q = Q^a = \epsilon(P, p, Y^a) \]
\[ q^b = \xi^b(p, Y^b) = \frac{Y^b}{p} \]

when the private \((q^a, q^b)\) consumption \((Q = Q^a)\) depends only one person’s income \((Y^a, Y^b)\) and the public \((Q)\) consumption only depends on one person’s income \((Y^a)\) instead of the incomes of two persons.

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The income pooling property generally does not hold when the private income is not summed up. This implies that generally the income distribution is important for household consumption in a non-cooperative environment. It is in each person’s self-interest to choose his/her action independently. However, the income pooling property holds when the private income is summed up such as case (1).

Then, the next obvious question we ask is whether non-cooperative approaches for case (1) and (2) are in the social interest/social efficient? To answer this, we turn our attention to private and socially efficiency below.

(3) Efficiency

In both cases, the non-cooperative approach leads to an inefficient outcome.

In case (1), the first order condition for a private allocation is

\[
\frac{\partial u^a}{\partial Q}(Q, q^a) = \frac{P}{p} = \frac{\partial u^b}{\partial q^b}(Q, q^b) \tag{30}
\]

In case (2), the first order condition for a private allocation is,

\[
\frac{\partial u^a}{\partial Q}(Q, q^a) = \frac{P}{p} \tag{31}
\]

where the value for \(\frac{\partial u^b}{\partial Q}(Q, q^b)\) is arbitrary and person b does not make any real decision on Q so that the marginal utility of public good for person b is indeterminate.

While the condition for an efficient allocation of public for case (1) and (2) come from utility

\[
L = \max_{Q, q^a, q^b} \lambda_a u^a(Q, q^a) + \lambda_b u^b(Q, q^b) + \mu[Y^a + Y^b - PQ - p(q^a + q^b)] \tag{32}
\]

This yields

\[
\frac{\partial L}{\partial Q} = \lambda_a \frac{\partial u^a}{\partial Q} + \lambda_b \frac{\partial u^b}{\partial Q} - \mu P = 0
\]

\[
\frac{\partial L}{\partial q^a} = \lambda_a \frac{\partial u^a}{\partial q^a} - \mu p = 0
\]

\[
\frac{\partial L}{\partial q^b} = \lambda_b \frac{\partial u^b}{\partial q^b} - \mu p = 0
\]

and the marginal rate of substitution between public and private goods is

\[
\frac{\lambda_a \frac{\partial u^a}{\partial Q} + \lambda_b \frac{\partial u^b}{\partial Q}}{\lambda_a \frac{\partial u^a}{\partial q^a}} = \frac{P}{p} = \frac{\lambda_a \frac{\partial u^a}{\partial q^a} + \lambda_b \frac{\partial u^b}{\partial q^b}}{\lambda_b \frac{\partial u^b}{\partial q^b}} \tag{33}
\]

where, in this social system, the marginal benefit is higher than in the private system. Giving up one unit of \(q^a\) gains one unit of Q for person a in the private system. In the social system, Giving up one unit of \(q^a\) (\(q^b\)) gains one
unit of \( Q \) for person \( a \) and one extra unit of \( Q \) for person \( b \). The extra utility effect on \( b \) is neglected in the private system.

Obviously, in the above cases (1) and (2), the opportunity cost in the private system is more (or less) than the willingness to pay for the public good so that individuals will want to contribute less (or more) than the social efficient level.

**Remark on the non-cooperative approach**

The properties of the equilibrium will differ according to which of the scenarios prevails. In general, there is no binding commitment or agreement among individuals and each has an independent preference. For the non-cooperative approach with no public goods, each has independent preference. The income pooling and the Slutsky conditions for household consumption generally do not hold (unless the consumers have Engel demand curves that are linear in income and have a common slope). For the non-cooperative approach with public goods, depending on whether each contributes to the public good or not, the possibility of the existence of income pooling property is low (unless the functional form of the household demand is special, meaning that household consumption is a linear function of total family income, \( Q_{NE} = \epsilon(P, p, Y^a + Y^b) \)) and both contribute to public goods without externalities. In both cases with public good, inefficiency problem arises.

Under the non-cooperative approach, each member has independent preferences and the income pooling property generally does not hold and income distribution is an important factor for family consumption.

**Cooperative model**  Cooperative models assume that individuals can communicate freely and make binding agreements. Such agreements yield benefits which can be shared among participants and solve the household’s resource allocation problem. Those models differ in the attention they pay to the question of how the household reaches this agreement. The following parts review three different types of models within cooperative environment.

**Collective model**  Browning and Chiappori (1998) claim that neoclassical utility theory applies to individuals but not to households, and they present a general characteristic of the collective model suggested originally by Chiappori (1988a and 1992). The two basic assumptions of the collective model are that each individual has his or her own preferences and the collective decisions are Pareto efficient. Consider a two person (A and B) household. Household purchases are associated with the market price vector \( p \). Household demands are divided into: private consumption by each person (\( q^A \) and \( q^B \)) and public consumption (\( Q \)). Household total demands are

\[
q^A + q^B + Q
\]
where the private good and the public good have the same unit. For example, \( q \) could be the private element of the telephone services and \( Q \) could be the public element of the telephone services.

The household budget constraint is

\[ p'(q^A + q^B + Q) = x \]  

(35)

where \( x \) is the total household income.

**Axiom 9** "Member i’s preference (I = A, B) can be represented as

\[ u^I(q^A, q^B, Q) \]

(36)

where \( u^I \) represents the ordinal preference and is strictly quasi-concave, non-decreasing and strictly increasing in at least one argument\(^{55}\). Each member’s utility depends on the other member’s consumption. This assumption allows for altruism, externalities or any other preference interaction.”

**Axiom 10** "The outcome of the household decision is Pareto efficient. For any price-income bundle \((p, x)\), the consumption vector \((q^A, q^B, Q)\) chosen by the household is such that no other vector \((q^{A'}, q^{B'}, Q')\) in the budget set could make at least one member better off”.

Let us now formally express the Pareto efficiency assumption. Household behavior, hence, must be a solution of the following program

\[ w^A(q^A, q^B, Q) \]

\[ st \]

\[ \mu : u^B(q^A, q^B, Q) \geq \pi \]

\[ \theta : p(q^A + q^B + Q) = x \]

for some utility level \( \pi \) which is an exogenous function of \( p, x \). The set of efficient outcomes obtains as, \( q^A(x, p), q^B(x, p), Q(x, p), \mu(x, p), \theta(x, p) \). The Lagrangian multiplier \( \mu(x, p) \) of the first constraint can be interpreted as the implicit weight of member 2’s utility in the collective decision. Therefore, the maximisation problem above is equivalent to the maximisation of \( w^A(q^A, q^B, Q) + \mu(x, p)u^B(q^A, q^B, Q) \) under the budget constraint \( p(q^A + q^B + Q) = x \). It is important to note that \( \mu \) will be a function of \( x, p \). If there exists an income-price dependent household welfare function \( W \), the household maximises the following program

\[ \max_{q^A, q^B, Q} W = \max_{q^A, q^B, Q} \{ w^A(q^A, q^B, Q) + \mu(x, p)u^B(q^A, q^B, Q) \} \]

\(^{55}\)This ensures all budget spends.
\[
p(q^A + q^B + Q) = x
\]

(1) The family (single stage) welfare maximisation problem can be written as

\[
\max_{q^A, q^B, Q} \{u^A(q^A, q^B, Q) + \mu(x, p)u^B(q^A, q^B, Q)\}
\]

\[+ \theta[x - p(q^A + q^B + Q)]\]

focs for \(q^A, q^B, Q\) are

\[
\begin{align*}
\frac{\partial u^A}{\partial q^A} + \mu \frac{\partial u^B}{\partial q^A} &= \theta p \\
\frac{\partial u^A}{\partial q^B} + \mu \frac{\partial u^B}{\partial q^B} &= \theta p \\
\frac{\partial u^A}{\partial Q} + \mu \frac{\partial u^B}{\partial Q} &= \theta p
\end{align*}
\]

so

\[
\frac{\partial u^A}{\partial q^A} + \mu \frac{\partial u^B}{\partial q^A} = \frac{\partial u^A}{\partial q^B} + \mu \frac{\partial u^B}{\partial q^B} = \frac{\partial u^A}{\partial Q} + \mu \frac{\partial u^B}{\partial Q}
\]

which satisfies Pareto efficient conditions for public goods and private goods with externalities.

(2) The optimal choices can also be solved in a two stage process. Person \(A\) and \(B\) live in a collective way, at stage 2, each decides their private consumption based on a private income distribution (person \(A\) gets income \(y^A\) and Person \(B\) gets income \(y^B\)). At stage 1, person \(A\) and \(B\) decide the income allocation and public provision.

**Stage 2:**

\[
q^A = \frac{y^A}{p}, q^B = \frac{y^B}{p}
\]

**Stage 1**

\[
\max_{y^A, y^B, Q} \{u^A\left(\frac{y^A}{p}, \frac{y^B}{p}, Q\right) + \mu(x, p)u^B\left(\frac{y^A}{p}, \frac{y^B}{p}, Q\right)\}
\]

\[+ \theta[x - y^A - y^B - pQ]\]

focs for \(y^A, y^B, Q\) are

\[
\begin{align*}
\frac{\partial u^A}{\partial q^A} \frac{\partial q^A}{\partial y^A} + \mu \frac{\partial u^B}{\partial q^B} \frac{\partial q^A}{\partial y^A} &= \theta \\
\frac{\partial u^A}{\partial q^B} \frac{\partial q^B}{\partial y^B} + \mu \frac{\partial u^B}{\partial q^B} \frac{\partial q^B}{\partial y^B} &= \theta \\
\frac{\partial u^A}{\partial Q} + \mu \frac{\partial u^B}{\partial Q} &= \theta p
\end{align*}
\]
where \( \frac{\partial u^A}{\partial q^A} = \frac{1}{p}, \frac{\partial u^B}{\partial q^B} = \frac{1}{p}. \)

So

\[
\frac{\partial u^A}{\partial q^A} + \mu \frac{\partial u^B}{\partial q^B} = \theta \n\]

we get

\[
\frac{\partial u^A}{\partial Q} + \mu \frac{\partial u^B}{\partial Q} = \theta p
\]

which satisfies the Pareto efficient conditions for public goods.

So long as the family has a "unitary" way of selecting goods with externalities or public goods, then the decisions on other private goods can be decentralised into two stages. This is true in a unitary model (p77) and in Browning & Chiapporis representation of public good choices.

**Axiom 11** "There exists a differentiable, zero degree-homogeneous function \( \mu(p, x) \) such that, for any \( (p, x) \), the vector \( (q^A, q^B, Q) \) are the solution to the program":

\[
\max_{q^A, q^B, Q} \mu(p, x) * u^A(q^A, q^B, Q) + [1 - \mu(p, x)] * u^B(q^A, q^B, Q) \]

subject to

\[
p(q^A + q^B + Q) = x
\]

where \( \mu(p, x) \in [0, 1] \) is the Pareto weight (or bargaining power) depending on the price and family income. Sometimes, \( \mu \) can be a function of the distribution factors \( Z \) (Browning et.al., 1994), that is \( \mu(p, x, Z) \). e.g. \( Z \) refers to the 'marriage market ' (Becker, 1991). This Pareto weight plays a critical role in the distribution of income within the household. If \( \mu(p, x) = 1 \), then the household behaves as though A always gets his way and A has a strong bargaining power. If \( \mu(p, x) = 0 \), then it is as if B is the dictator of the household. For interior values, A and B, each have some decision power.

The above household utility looks a lot like the BSWF but with one extra term - the Pareto weight \( (\mu(p, x)) \), that is

\[
\max_{q^A, q^B, Q} W(u^A(q^A, q^B, Q), u^B(q^A, q^B, Q), \mu(p, x))
\]

\[
= \max_{q^A, q^B, Q} W(u^A(q^A, q^B, Q), u^B(q^A, q^B, Q), p, x)
\]
However, the collective model is different from the BSWF and the critical feature of the collective model is that it depends on the Pareto weight $\mu(p, x)$. This property is different from the BSWF, in which the household behaves like an individual, household members have a mutual agreement on the income distribution and there is an invisible hand which allocates family resources to each member. After receiving the resource, each maximises utility subject to the resource constraint. While, in collective model, depending on the Pareto weight, household can either behave as an individual or as many individuals. The income distribution is determined by this Pareto weight. In particular, the distribution can depend on the price vector. For example, if the cost of child care is high, then the person who spends most of his/her time on child care might get a high income allocation.

The Pareto weight is important for two reasons. First, an efficient household does not need to behave like an individual satisfying the standard conditions of consumer theory (e.g. income pooling). Second, it introduces prices into a household utility function, in the sense that prices enter the Pareto weight, affecting the respective weights of individual utilities in the household welfare function. But prices do not enter the individual utility functions.

A geometric interpretation (Fig 40) of the Pareto weight is explained here. Let’s suppose $W = \mu u^A + (1-\mu) u^B$ and budget constraint is $PQ + p(q^A + q^B) = x$, then the set of all pairs $(u^A, u^B)$ satisfying the collective maximization problem (maximise $W$ st the budget constraint) is the set of all efficient allocation ($UPF(P, p, x) = \{u^A, u^B | u^A = u(q^A, Q), u^B = u(q^B, Q), PQ + p(q^A + q^B) = x\}$). It is known as $UPF$ (utility possibility frontier) or as the Pareto frontier. And $\mu(P, p, x)$ defines the location of the final outcome of the allocations. In figure 2, (i) a shift from point I to point II results from the change in price ($P$ or $p$) and income ($x$). But $\mu(P, p, x)$ stays constant in this shift. And the $UPF$ is also a different one ($UPF(P', p', x') = \{u^A, u^B | u^A = u(q^A, Q), u^B = u(q^B, Q), P'Q + p'(q^A + q^B) = x'\}$).
(ii) A shift from point II to point III in the same UFP is caused by the change in Pareto weight. But the budget constraint stays the same. The second shift is the collective effect. (iii) A shift from point I to point III in the different UFP is caused by the change in Pareto weight, price ($P$ or $p$) and income ($x$).

**Remark on collective model**

- The collective model looks like the unitary model but is different from the unitary model. The critical difference lies in the Pareto weight and the collective effect.
- The preferences in the collective model are not restricted. The individuals can be either egoistic or altruistic.

In an example of the altruistic case, the utility of $A$ and $B$ can be expressed as

$$
U^A(q^A, q^B, Q) = u^A(q^A, Q) + \delta^A u^B(q^B, Q) \tag{40}
$$

$$
U^B(q^A, q^B, Q) = u^B(q^B, Q) + \delta^B u^A(q^A, Q)
$$

where $\delta^h (h = A, B)$ is the degree of altruism

The household decision problem is now

$$
\max_{q^A, Q, q^B} W = \mu * [U^A(q^A, q^B, Q)] + (1 - \mu) * [U^B(q^A, q^B, Q)] \tag{41}
$$

$$
= \mu * [u^A(q^A, Q) + \delta^A u^B(q^B, Q)] + (1 - \mu)[u^B(q^B, Q) + \delta^B u^A(q^A, Q)]
$$

$$
= (\mu + \delta^B - \mu \delta^B)u^A(q^A, Q) + (1 + \mu \delta^A - \mu)u^B(q^B, Q)
$$

- The children can be considered as public goods or independent decision makers. In the first case, if a couple (A and B) both care about the kid, then the household utility becomes

$$
\max_{q^A, Q, q^B} W = \mu * (u^A + \kappa^A u^{kid}) + (1 - \mu) * (u^B + \kappa^B u^{kid}) \tag{42}
$$

where $\kappa^h (h = A, B)$ is the degree of altruism given by each person.

In the second case, children have their own decision power and household utility becomes,

$$
\max_{q^A, Q, q^B} W = \mu * u^A + (1 - \mu - \kappa^{kid}) * u^B + \kappa^{kid} u^{kid} \tag{43}
$$

- The Slutsky matrix generated by the neoclassical utility model and collective model are also different. For example, in the neoclassical utility model, the household’s program is $\max_{X^A, X^B} W(u^A(X^A), u^B(X^B))$ subject to $p^A X^A + p^B X^B = Y^A + Y^B = Y$
where $X^A = X(p^A, p^B, Y); X^B = X(p^A, p^B, Y)$.

the Slutsky matrix is

$$
S_{p^A, Y} = \frac{\partial X^B}{\partial p^A} + X^B \frac{\partial X^B}{\partial Y}
$$

$$
S_{p^B, Y} = \frac{\partial X^A}{\partial p^B} + X^A \frac{\partial X^A}{\partial Y}
$$

while the Slutsky matrix for the collective model ($X^A = X(p^A, p^B, Y, \mu(p^A, p^B, Y)); X^B = X(p^A, p^B, Y, \mu p^A, p^B, Y)$) is

$$
S_{p^A, Y} = \left( \frac{\partial X^B}{\partial p^A} + X^B \frac{\partial X^B}{\partial Y} \right) + X^B \left( \frac{\partial X^B}{\partial \mu} + \frac{\partial X^B}{\partial Y} \right)
$$

$$
S_{p^B, Y} = \left( \frac{\partial X^A}{\partial p^B} + X^A \frac{\partial X^A}{\partial \mu} \right) + X^A \left( \frac{\partial X^A}{\partial Y} + \frac{\partial X^A}{\partial \mu} \right)
$$

The extra collective effect generalizes the different substitution and income effect of the unitary neoclassical model. This collective effect is influenced by the price.

**Nash bargaining**

Nash (1950) presents a two-person bargaining situation in which there is a conflict of interest about agreement; individuals have the possibility of conducting a mutually beneficial agreement, and no agreement can be imposed on any individual without his approval. In Nash’s work, the threat point is exogenous, every one is in a equal position for the agreement, each individual wishes to maximize the utility to himself of the ultimate bargain, each have equal bargaining skill. He shows that any allocation consistent with a set of axioms in his paper then must maximize the Nash product of utilitygains above the threat point wrt each individual’s choices,

$$
\max_{x^1, x^2} [u^1(x^1) - v^1][u^2(x^2) - v^2]
$$

s.t. : $p(x^1 + x^2) = y^1 + y^2$

where $v^i(\cdot)$ ($i = 1, 2$) is the utility obtained if one decides not to bargain with the other player; $v^i$ can be interpreted as the threat point; $p$ is the price vector of commodity $x$; $y^i$ is each individual’s income. The product of the two excess utilities is generally referred to as the Nash product function.

**Nash bargaining with threat point (Divorce-threat bargaining model)**

We use the Nash bargaining model with a threat point from the paper of McElroy and Horney (1981). In the paper, they assume that a household consists of two people, husband (m) and wife (f). They live in a cooperative
way but each has divorce-threat point. \( T^h, h = m, f \). The divorce-threat point is defined as the maximal level utility attainable outside the marriage. In particular, \( T^h \) is the indirect utility function corresponding to the separate maximization of individuals’ utilities subject to their own budget constraints: \( T^h = V^h(p^h, y^h, P, \alpha^h) \). \( \alpha^h \) corresponds to the relevant shift parameters such as the opportunities outside of the marriage change (e.g. the \( \alpha^h \) might be a matching rate in the marriage market. If the matching rate is high for \( h \), then person \( h \) might have a high threat point because of good opportunities outside the marriage ). The married couple maximise the Nash product function

\[
\max_{X^m, X^f, Q} \left[ u^m(Q, X^m) - V^m(p^m, y^m, P, \alpha^m) \right] \left[ u^f(Q, X^f) - V^f(p^f, y^f, P, \alpha^f) \right]
\]

subject to

\[
PQ + p^m X^m + p^f X^f = y^m + y^f
\]

where \( y^h \) consists of the wage income and non-wage income \( I^h \); \( X^h \) are private consumptions at price vector \( p^h \); \( Q \) are the public goods of the household and \( P \) is the relative price vector.

Remark:

- The Nash product function derived in McElroy and Horney’s (1981) model is different from the neoclassical utility model. In particular, in the neoclassical model the changes in prices and income change the optimal consumption bundle via the shifts in the budget constraint while the utility remains fixed. In the divorce-threat bargaining model, the changes in prices and incomes not only shift the budget constraint but also the objective function by changing the bargaining power via the divorce-threat point \( (T^h = V^h(p^h, y^h, \alpha^h) ; \text{premarital indirect utility}) \).

- Also, McElroy and Horney (1981) leave an open question on how the non-wage income affects the threat point, although property rights in this income are likely to be important.

Nash bargaining with public good and intrahousehold transfer

We discuss a model of intra-household transfer through a family bargaining process from Chen & Woolley (2001). There are two special properties of the model in Chen & Woolley (2001) paper: (1) unlike the model of McElroy and Horney (1981), the Nash equilibrium solution (e.g. contribution to public good) is sensitive to non-wage income, namely intrahousehold transfers (the changes in division of family income will in general change expenditure on the public good); (2) the final public consumption \( (x^h) \) allocation is not Pareto efficient. Chen & Woolley (2001) present a sequential game with two adults (for \( i = m, f \), male and female), two private goods \( (x_i, x_j) \) with price
and one public good $x^h$ with price normalised to unity (each contributes to this public good $x^h = x^h_i + x^h_j$) and two sided altruism. The model developed in their paper contains a two stage game. The reasoning process is backward induction, from the end of a problem, to determine a sequence of optimal actions. In the second stage, given income (e.g. $y_f = I_f + \pi$), intrahousehold transfer (e.g. $\pi$ sent from $m$ to $f$) and the amount of the other’s contribution to public good (e.g. $x^h_m$), individual $i$ makes decisions independently on private and public consumptions (e.g. $x_f, x^h_f$). In the first stage, given purchase plans from $m$ and $f$ of the second stage, the family jointly determines intrahousehold transfers ($\pi$). Chen & Woolley analyse three cases: no transfer; voluntary income transfer; Nash-bargained income transfers. We mainly look at the third case, Nash-bargained income transfers.

In the second stage: the $x_i, x^h_i$ are decided through the individual’s optimization problem,

$$\max_{x_i, x^h_i} W_i = U_i + sU_j$$

$$= [u(x_i) + v(x^h_i + x^h_j)] + s[u(x_j) + v(x^h_i + x^h_j)]$$

subject to budget constraint,

$$y_f = (I_f + \pi) = x^h_f + px_f$$

or, $y_m = (I_m - \pi) = x^h_m + px_m$

where $W_i = U_i + sU_j$ is each individual’s welfare function (two way altruism, $i$ is altruistic towards $j$ and $j$ is altruistic towards $i$); $s \in [0, 1]$ is the degree of altruism to the partner; $I_i$ is wage income; $y_i$ includes wage and non-wage income (transfer).

Solutions give reaction curves,

$$x_i = x_{i,RC}^h(x_j, x^h_j, p, I_i, \pi, s)$$

$$x^h_i = x_{i,RC}^h(x_j, x^h_j, p, I_i, \pi, s)$$

Through this, we can see the property (1) that "the changes in division of family income will in general change expenditure on public good".

and NE

$$x_i = x_{i,NE}^h(p, I_i, \pi, s)$$

$$x^h_i = x_{i,NE}^h(p, I_i, \pi, s)$$
In their paper (Chen & Woolley, 2001), they claim that "the resource allocation is, in general, not Pareto efficient" we elaborate this here. For simplicity, we only look at the private and social optimal conditions for public good consumption.

Private system,

\[
\max_{x_f^h \geq 0} W_f = \left[ u_f(x_f) + v_f(x_f^h + x_m^h) \right] + s[u_m(x_m) + v_m(x_f^h + x_m^h)] + \mu[I_f + \pi - x_f^h - px_f] \\
Foc, x_f^h : \frac{-1}{p} u_f' + v_f' + sv_m' = 0
\]

\[
\max_{x_m^h \geq 0} W_m = \left[ u_m(x_m) + v_m(x_f^h + x_m^h) \right] + s[u_f(x_f) + v_f(x_f^h + x_m^h)] + [I_m - \pi - x_m^h - px_m] \\
Foc, x_m^h : \frac{-1}{p} u_m' + v_m' + sv_f' = 0
\]

Social system,

\[
\max_{x_f^h \geq 0, x_m^h \geq 0} L = (1 + s)[u_f(x_f) + v_f(x_f^h + x_m^h)] + (1 + s)[u_m(x_m) + v_m(x_f^h + x_m^h)] + \mu[I_f + I_m - p(x_f + x_m) - (x_f^h + x_m^h)] \\
Foc, x_f^h : (1 + s)[\frac{-1}{p} u_f' + v_f'] + (1 + s)v_m' = 0 \\
\hspace{1cm} : \frac{-1}{p} u_f' + v_f' + v_m' = 0 \\
Foc, x_m^h : \frac{-1}{p} u_m' + v_m' + v_f' = 0
\]

The above focs from equations (53), (54) and (55) suggest that the final public consumption allocation is Pareto efficient only when \( s = 1 \). Otherwise, the final public consumption allocation is not Pareto efficient. This supports the second property claimed above.

**Remark:**

- Besides the two special properties claimed above, the decision process for public good is also different. Each individual spends their incomes rationally, they choose their expenditure on the household public good non-cooperatively. Then a obvious question arises: why do both partners follow this two-stage process of a cooperative income allocation and then a non-cooperative expenditure allocation? Why do not they just bargain directly on consumption choices and not on income shares and then they might be both better off?
4.2.2 OLG

In a society or in a household, social interactions or economic interactions are a necessary part of individuals’ life. Those interactions might affect individuals’s economic decisions such as savings, education investment and consumption. The overlapping generation model is a framework that explicitly explains those interactions between generations and allows a richer pattern of economic decisions and allows us to analyse different forms of redistribution. The models presented here come from Samuelson (1958) and Diamond (1965).

We start our analysis from Samuelson (1958) and then add the firm behavior into OLG (Diamond, 1965).

**Samuelson** Samuelson (1958) introduces the overlapping generations model in a discrete time setting. In each period, a new generation is born and lives for three periods. Each produces one unit of output in period 1 and one unit in period 2, in period 3 he retires and produces nothing. Each person’s tastes can be expressed by an ordinal utility function of the consumptions of the three periods of his life: \( U = U(C_1, C_2, C_3) \). This is the same for every generation.

Samuelson (1958) proposes a consumption-loan idea, that is, the working generations consume part of the output and give the remaining parts \((1 - C_1, 1 - C_2)\) as free gifts to the non-working generation who produces zero output. But there are problems of incentives for giving. If the product is non-storable, then each selfish generation will consume everything and leave zero for the non-working generation. If the product is storable, then each selfish generation will prefer to save the remaining part for himself instead of giving it away as a gift. Suppose the product is storable at a constant interest rate \( i \) such as an exogenous natural growth rate for yoghurt or a decay rate for perishable food, then there is no incentive for an individual to make a free gift unless the saving interest rate is zero and the individual cannot consume the one unit of product entirely. Otherwise, the selfish individual will prefer to store good intertemporally for himself/herself (or to consume everything in one period).

In Samuelson’s 1958 paper, he argues that the incentives problem cannot be solved without a social security system or family altruism assumption. We review his arguments in terms of private optimum and social optimum below to compare the differences between private optimum and social optimum and to understand why working individuals have no interest in giving free gifts to the non-working generations without a social security system or family altruism assumption.

First we look at the private optimum below,

**Private optimum**
In Samuelson’s (1958) paper, a representative individual is thought to maximise his lifetime utility

\[
\max_{C_1, C_2, C_3} U(C_1, C_2, C_3)
\]  

(56)

subject to lifetime budget constraint

\[
C_1 + \left(\frac{1}{1+i}\right)C_2 + \left(\frac{1}{1+i}\right)^2C_3 = 1 + \left(\frac{1}{1+i}\right) \ast 1 + \left(\frac{1}{1+i}\right)^2 \ast 0
\]  

(57)

\[
C_1 + RC_2 + R^2C_3 = 1 + R + 0R^2
\]

where \(C_1, C_2, C_3\) are consumptions over three periods; here, we can interpret \(i\) as a decay rate; \(R = \frac{1}{1+i}\) is the constant discount factor between goods of period \(t\) traded for goods of the next period \(t+1\). If \(R = 1\), the discount factor is very high (the interest rate \(i\) is zero) and tomorrow’s good costs 1 of today’s good (one to one physical transfer of goods intertemporally).

Through the optimisation problem, we get optimal choices for \(C_1, C_2, C_3\) in private system which satisfy the conditions below,

\[
\frac{\partial U}{\partial C_2} = \frac{\partial U}{\partial C_3} = R = \frac{1}{1+i}
\]

(58)

where the marginal rate of substitution of consumption equals to the discount factor.

**Social optimum**

Above, the problem of the individual is solved in the decentralized system. Below, we review the optimal choices for consumptions in a social system. Suppose there is a social planner, he allocates resources between the three generations living in the same time period \(t\) under a social welfare function,

\[
\max_{C_{1,t}, C_{2,t}, C_{3,t}} U(C_{1,t}, C_{2,t}, C_{3,t})
\]

(59)

subject to

\[
C_{1,t} + \frac{1}{1+m}C_{2,t} + \frac{1}{(1+m)^2}C_{3,t} = 1 + \frac{1}{1+m}
\]

(60)

where \(C_{1,t}, C_{2,t}, C_{3,t}\) are consumptions for generations 1 (young), 2 (middle aged) and 3 (the elderly) at \(t\); \(m\) is the constant population growth rate. In a growing population the age distribution is in favor of the younger productive ages, so the society or each household has an age distribution proportional to \([1, \frac{1}{1+m}, \frac{1}{(1+m)^2}]\) (the number of young population is larger than that of the middle aged and the elderly ones).

Then we get

\[
\frac{\partial U}{\partial C_{2,t}} = \frac{\partial U}{\partial C_{3,t}} = \frac{1}{1+m}
\]

(61)

where the social planner chooses optimal \(C_{1,t}, C_{2,t}, C_{3,t}\) when the marginal rate of substitution equals to the population growth factor. For instance, the marginal cost of transferring one unit of \(C_{1,t}\) to \(C_{2,t}\) equals to \(\frac{1}{1+m}\) while
the marginal cost of transferring one unit of $C_{2,t}$ to $C_{1,t}$ is much higher $(1 + m)$. As the population of young generations is larger than that of the middle aged generations, the cost of moving resources from generations 1 can be shared by more people.

**Efficiency**

Now we move to efficiency problem to see when the private optimum and social optimum effectively coincide for optimal consumptions. We basically set the marginal rates of substitution in the social system and in the private system to be equal,

$$\frac{\partial U/\partial C_{2,t}}{\partial U/\partial C_{1,t}} = \frac{\partial U/\partial C_{3,t}}{\partial U/\partial C_{2,t}} = \frac{\partial U/\partial C_{2}}{\partial U/\partial C_{1}} = R = \frac{1}{1 + i}$$

then we get

$$\frac{1}{1 + m} = R = \frac{1}{1 + i}$$

where the private optimum and social optimum effectively coincide when the population growth rate $m$ equals to the decay rate $i$. If this condition holds, then the social optimum can be achieved through the private first order conditions. However, this addresses two difficult issues. First, $m$ and $i$ are both exogenous and the social planner cannot automatically set those two values and let them to be equal. This mission is impossible for a society to achieve. Second, there are still incentive problems. Why should the working generation give free gifts to the non-working generations? Unless there is a guarantee that those working generations will be supported in the future when they are in the non-working situation without any savings. We need a system to guarantee this. Thus, Samuelson (1958) proposes two possible solutions: a social security (central planner) system or family altruism. Such a condition cannot hold without those two. Otherwise, the aged have no claims on the young. "The cold and selfish competitive markets will not teleologically respect the old, the aged will get only what supply and demand impute to them". In Samuelson’s (1958) paper, he fully analyses the social security idea but briefly mentions family altruism idea and leaves it as an open box. This raises our interest in family altruism in this chapter.

**Remark:**

- Given the condition that the young generation is continuously coming, the social security system or social collusion system can work and the social optimum can be achieved. If there is a central planner, this central planner brings all generations into the social contract via a tax system, in which the young generation needs
to obey the regulation to financially support the old by paying tax, then the social security system will work. Such support is guaranteed by a draft on the yet-unborn, then the young generation will be financially supported in their old age by the new young. This income tax becomes in part a device to prevent individuals from defaulting on giving transfers to the old. That is, if the young deviate from the contract, then they will not receive support from the central planner. This tax system ensures that each young generation is fully committed to send free gift/money to the old generation.

- The social security idea has been well developed but the family altruism idea still leaves "an open box". Instead of implementing regulation and law, can family altruism put men into binding social contracts? In a society, if each household lives in a love environment, then can the social optimum be achieved within the small unit of a family? How will this altruistic system work within a household? Who is the altruistic person? What are the incentives for giving? Will the private and social optimal both be achieved? Those are interesting questions for us and we will try to answer those questions in this chapter later.

- Further, the OLG could be expanded to consider the technological investment possibilities, intertemporal investment, innovation and various aspects of uncertainty.

Diamond Diamond (1965) adds capital and a production function which combines Samuelson’s OLG with a growth model. In Samuelson’s model, the interest rates are exogenously determined. While, in Diamond’s model, the interest rates are determined by the marginal product of capital.

Diamond considers a two-period (youth and retirement) OLG in a time discrete setting. In period 1, the working young provide the labor and save; in period 2, the old retires and gets dividend from saving.

**Private optimum**

The individual’s maximization problem is

\[
\max_{C_t^Y, C_t^O, S_t} u(C_t^Y, C_t^O) \tag{63}
\]

subject to

\[
C_t^Y + S_t = w_t \tag{64}
\]

\[
C_{t+1}^O = (1 + r)S_t
\]

where \(S_t\) is saving; \(w_t\) is the wage rate with one unit labor supply; \(r\) is the interest rate. Thus the utility maximization yields the consumption Euler equation,

\[
\frac{\partial u}{\partial C_{t+1}^O} = \frac{1}{1 + r} \tag{65}
\]
Firm

Diamond (1965) adds a firm production function into OLG model. This firm employs labor from the young generation and attracts capital from the old generation. Thus the aggregate production is

\[ Y_t = F(K_t, L_t) \]  
(66)

where \( K_t \) is the aggregate capital stock from the old generation and \( L_t \) is the aggregate labor input of the young generation.

In a competitive market, the firm exhibits its maximisation problem as follows,

\[ \max_{K_t, L_t} F(K_t, L_t) - wL_t - rK_t \]  
(67)

Through the maximisation problem, the interest rate of saving equals to the marginal product of capital and the wage rate equals to the marginal product of labour, thus

\[ r = \frac{\partial F(K_t, L_t)}{\partial K_t} \]  
(68)
\[ = F_{K_t}(K_t, L_t) \]
\[ w = \frac{\partial F(K_t, L_t)}{\partial L_t} \]  
(69)
\[ = F_{L_t}(K_t, L_t) \]

Social optimum

Assuming that all the individuals have the same lifetime consumption pattern, if there is a social planner, then the social planner at time \( t \) allocates consumptions between the young and the old generations,

\[ \max \Lambda = u(L_tC_t^Y, L_{t-1}C_{t-1}^O) \]  
(70)

where \( L_tC_t^Y \) and \( L_{t-1}C_{t-1}^O \) are the aggregate consumptions for the Y and the O at time \( t \).

subject to the aggregate budget constraint

\[ F(K_t, L_t) - (K_{t+1} - K_t) = L_tC_t^Y + L_{t-1}C_{t-1}^O \]  
(71)
\[ F(K_t, L_t) - nK_t = L_tC_t^Y + L_{t-1}C_{t-1}^O \]

where, \( K_{t+1} = (1 + n)K_t \)

where \( n \) is the population growth rate; if \( n = 0 \), the capital stock \( K_{t+1} \) at time \( t + 1 \) equals to the stock at \( t \), \( K_t \).
Given that,

\[ L_t = (1 + n)L_{t-1} \quad (72) \]

the aggregate budget constraint can be written

\[ F\left( K_t, L_t \right) - nK_t = L_tC_t^Y + \frac{L_tC_t^{O-1}}{1 + n} \quad (73) \]

Denote everything in per-capita units - divided by \( L_t \)

\[ F\left( \frac{K_t}{L_t}, \frac{L_t}{L_t} \right) - \frac{nK_t}{L_t} = \frac{L_tC_t^Y}{L_t} + \frac{L_tC_t^{O-1}}{(1 + n)L_t} \quad (74) \]

where \( F\left( \frac{K_t}{L_t}, \frac{L_t}{L_t} \right) \) can be written as \( f(k_t) \); \( f \) is the production function of capital per person.

Then, the above equation can be written as,

\[ f(k_t) - nk_t = C_t^Y + \frac{C_t^{O-1}}{1 + n} \quad (75) \]

where at time \( t \) the output equals to the sum of the consumption of the \( Y \), of the \( O \) and the saving.

From

\[ \max u(C_t^Y, C_t^{O-1}) \quad (76) \]

\[ \text{s.t : } F\left( \frac{K_t}{L_t}, \frac{L_t}{L_t} \right) - \frac{nK_t}{L_t} = C_t^Y + \frac{L_tC_t^{O-1}}{L_t} \]

we get

\[ (a) : \frac{\partial u}{\partial C_t^{O-1}} = \frac{L_t}{L_t} = \frac{1}{1 + n} \quad (77) \]

where the optimal consumption can be achieved when the marginal rate of substitution equals to \( \frac{1}{1+n} \). The consumption is smoothed when \( n = 0 \).

Through the aggregate resource, the marginal product of capital per person equals to the population growth rate.

\[ \frac{f'(k_t)}{f''(k_t)} \left( \frac{K_t}{L_t}, \frac{L_t}{L_t} \right) = n \quad (78) \]

\[ f'_{k_t}(k_t) = n \]

Given that marginal product of capital per person equals to the interest rate in the private system, then the social optimum and private optimum coincide when

\[ \frac{\partial u}{\partial C_t^{O-1}} = \frac{1}{1 + n} = \frac{\partial u}{\partial C_t^{O-1}} = \frac{1}{1 + r} \quad (79) \]
\[ F_{Kt} (Kt,Lt) = F'_{Kt} \left( \frac{Kt}{Lt} \right) = f_{Kt}^' (k_t) = r = \frac{f_{Kt}^' (k_t)}{n} \]

where \( r = n \), the interest rate equals to the population growth rate.

**Efficiency**

Therefore, we say that the social optimum can be achieved through the private system when

\[ r = n \]  

**Remark:**

- The social optimum can be achieved when the golden rule for optimal production (78) and optimal consumption (77) are both satisfied. This requires interest rate \( r \) equals to the population growth rate \( n \). Since \( n \) is exogenous but \( r \) is endogenous (equals to MPK), there is no guarantee that \( r = n \). Diamond (1965) applies the central planner method and suggests the government debt idea to solve the inefficiency problem especially when \( r < n \). The government issues bonds at market interest rate \( r \) so that now part of saving goes into bond market instead of only capital market (Saving=Bond+capital). The firm owns all the capital in the capital market and the government acts as the central planner to transfer wealth from young to old and lowers the utility of young.

- Family altruism is also not mentioned by Diamond (1965). In this static setting, if the altruistic young respects the old and does high savings as soon as the utility of the old is decreasing (or the consumption level is dropping), then the altruistic behavior might play the same role as the government bond does. The young send the saving as a free gift to the old. If each generation is fully committed to give free gifts, then the current young generation will send free gifts to the current old generation and will receive free gifts from their future young generation in the next period. As there is infinite future young generation comes, then the system won’t break up and each young generation will happy to support the old generation.

- But the conditions for private optimum and social optimum in a dynamic setting with altruistic behavior have not been studied yet. Also, can family altruism solve the inefficiency problem? If not, do we still need a central planner to obtain the efficiency? All these interesting research questions have not been answered, this chapter tries to answer those questions as well as we can.
4.2.3 The type of altruism

When people make donations or transfers to the public or to others, such as charity and bequest, there may be many factors influencing their decisions.

**Pure altruism**  Gary Becker (1974) first introduced the altruism/sympathy idea to analyze the altruistic incentives. He explains that a redistribution of income from one family member to the others is motivated by pure caring. For example, transfers from parents to children in the form of education, free gift and bequest because the parents love their children truly and they care about the well-being of the children. This altruism is normally understood to denote a preference which does not rely on any social norm or social expectations but on the well-being of others, which is considered as unconditional altruism (Konow, 2006).

There are two possible scenarios for this type of caring. In one case, the donor respects the preferences of the recipient and takes the preferences of the recipient as given. In the second case, the donor generally cares about the welfare of the recipient but disagrees with the recipient’s preferences and acts on his view of desirable recipient preferences. In this chapter, we mainly consider the first case.

Nevertheless, this aspect of theoretical work on altruism is not the only stream.

**Censored altruism**  Andreoni (1989, 2006) adds ‘warm glow’ to explain the motivation of giving, Olson (1965) mentions social respect as one incentive for transfers within a group and Sen (1983) discusses about social norms/expectations for the altruistic behavior. Explicitly, Andreoni (1989, 2006) considers ‘warm glow’ as one motive for altruistic behavior. He believes that giving brings a pleasure feeling for the donor and defines this warm glow as impure altruism. And this motive is formulated as the giver receives the positive benefit and is distinct from the positive benefit enjoyed by the recipient in pure altruism. Also, Olson (1965) mentions in his book that the collective behavior is explained not only by the economic incentives but also by the social respect, friendship and other social objectives. In addition, Sen (1983) calls the altruism which is formalized as a social norm/respect/objectives as coerced altruism. Therefore, in our paper we name the second stream of altruism as censored altruism which is motivated either by a pleasure feeling (warm glow) or by the social respect or by the social norm/expectation (coerced altruism). And the preferences of these three different motivations (warm glow, social respect and social norm/expectation) have strong similarities as the preferences of the donor rely on some private benefit to the donor but not the well-being of the receiver. Further, Alger and Weibull (2007) argue that the motive for sharing resources within a household is a mixture of sympathy and commitment, or is a mixture of voluntary and coerced altruism, or is a mixture of pure altruism and censored altruism.
Semi-altruism  In the 1974 paper, Becker did not distinguish the different types of altruism so clearly. The altruism in this paper could be interpreted as pure altruism (inter-dependent dynamic well-being) or semi-altruism (inter-dependent static well-being). We extend Becker’s altruism to semi-altruism, which is between pure altruism and censored altruism. In our paper, we define semi-altruism as bounded rationality or myopic love in which the preference of the donor depends on the current well-being of the recipient but the well-being of the recipient in the future periods is ignored. In the semi-altruism model, altruistic individuals are bounded rational or myopic, who only know the current preference of the recipients. Because of the imperfect foresight, the donor does not really know the preferences of the recipients in the future periods and he only observes the current preferences of the recipients. e.g. the altruistic grandparents can only see the happiness of the grandchildren when grandparents are still alive.

4.3 The theoretical framework

Above, we review the literature for household modelling, OLG and three types of altruism. From the review, the social security idea has been well developed but the family altruism idea still leaves "an open box". Instead of implementing regulation and law, can family altruism put men into binding social contracts within the family? In a society, if each household lives in a love environment, then can the social optimum be achieved within the small unit of a family? This "open box" leaves us a variety of other research questions. For example, how will this altruistic system work within a household? How could the altruistic behavior be realized? Who is the altruistic person? What are the incentives for giving? Will the private and social optimal both be achieved. In addition, knowing that the household consists of individuals exhibiting different preferences, the unitary model cannot be applied but the alternative household model (non-cooperative household model) could be applied to study heterogenous individual preferences. Further, the OLG could be expanded to consider intertemporal education investment to discuss the effect of the altruistic behavior on the education investment.

These concerns and interests have led us to develop an analytical framework to understand these issues. In particular, can altruistic behavior and intergenerational transfers be the private vehicles to achieve a social/Pareto optimum? Following this, this section includes the following parts as: (i) an introduction for the basic setting of the theoretical framework; (ii) a variety of main concepts such as individual decisions and private Nash equilibrium and a section to understand whether the private NE achieve social optimum; (iii) some possible corrections if private equilibrium cannot be social optimum.
4.3.1 The introduction of the economic environment

We consider an OLG in a discrete time setting. In each period, a new young person is born and lives for three periods. In each time period, three generations (O: old (grandparent); M: middle aged (parent); Y: young (grandchild)) live in the same household (Fig 41). For example, at time $t$, the Y (born at $t$), the M (born at $t - 1$) and the O (born at $t - 2$) are living in the same household (see the dotted rectangle in Fig 41). The lifetime transfers and decisions for one agent (e.g. the Y born at $t$) can be outlined as this (see the solid rectangle in Fig 41): in period 1 (e.g. at time $t$): each agent is assumed to be dependent when he/she is still young, and then becomes an independent adult when he/she turns to be M; each M is assumed to be altruistic towards the Y and the O; the M sends a free gift to the O (e.g. $R_{t,t-1}$, where $t$ is the current time and $t - 1$ is the time of birth) and makes education investment on the dependent Y (e.g. $T_{t,t}$, where the first $t$ is the current time and the second $t$ is the birth date for the Y) and decides the amount of saving for himself/herself ($S_{t,t-1}$).

The incentive for giving free gifts or doing human capital investment is altruism. The altruistic individuals derive their utility from their own consumption and their one-way altruism ($M \rightarrow O, M \rightarrow Y$). Furthermore, for the purpose of understanding the role of intra-household transfers, we assume that the uneducated Y and retired O do not work and the M is the only working generation. In addition, we extend one way altruism to reciprocal altruism in the sense that the altruistic agents will be in a reverse situation where the person whom he helped before will perform an altruistic act towards him. Each middle aged person of any generation is assumed to have an identical degree and form of altruism towards the young and the old alive at that date.

If there are N agents born at at each date, then the framework also describes an economy with a large number of agents.

As indicated in the earlier literature, there could be three types of scenario in which the inter-generational wealth flows are motivated by warm glow (Andreoni, 1989, 2006), semi-altruism (Becker, 1974) or pure altruism (Bernheim, 1989) respectively. The differences between these three scenarios lies in the explanation of why altruistic agent makes free gift and finances HC investment of others. (i) In the censored altruism case, the reason is down to some social norm or custom or self-reputation which compels the M to do so, irrespective of what the recipients want. (ii) In the semi-altruism case, the reason is explained by the bounded rationality or myopic love in which the preference of the donor depends only on the current observable utility (or the current consumption) of recipients. The donor cannot foresee the recipients future state. (iii) In the pure altruism case, the reason is due to pure love as

\[ R_{t,t-1} \]

For most variables, we use two subscripts. The first subscript is the current time and the second subscript is the birth date. For instance, $R_{t,t-1}$, where the first $t$ is the current time and the second $t - 1$ is the birth date. This action is done by the M at time $t$ who was born at time $t - 1$. 

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the M cares about the lifetime utilities of others and can foresee these utilities. In the pure altruism case, the current middle aged cares the wellbeing of all the future generations through the altruism that they have their children at an infinite horizon. This leads to an infinite horizon problem. There is an infinite number of lifetime utilities of all future generations which enter the lifetime utility of the finite lived altruistic generation. Then the private lifetime utility will not be bounded. To solve this problem, we assume lifetime utilities are time additive to break down the link between the current altruistic generation and the nonadjacent future generations. In this chapter, we build a framework based on the idea of warm glow and study whether the private system can attain an efficient outcome. If not, some possible corrections, such as pure altruism (semi-altruism is one special case of pure altruism, so here we just use pure altruism for our policy correction) and taxation will be suggested.

We assume that each agent has perfect foresight on the others’ choices and everyone is fully committed. This full commitment assumption can be justified as follows: we believe that the habit or preference of each young generation is formed through the family environment. The behavioral patterns formed by the middle aged are literally etched in the young generations’ neural pathways. If the middle aged deviate from the altruistic contract with the old by not sending any free gifts, then the young generation will copy the same behaviour and won’t send any free gifts to the middle aged when the middle aged turns to be old. In order to prevent the young generation from copying the same bad habit, the current middle aged will behave well by sending free gift to the old which helps the young generation form a good habit. Through this, each generation will form a good habit being altruistic towards the old. Therefore, each middle aged generation will commit to the altruistic contract with the old. Meanwhile, using the same logic, each middle aged generation does educational investment on the young generation as well. Through out,

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57(1) "Children are three times more likely to spend lots of time watching TV and playing on screens if their parents do the same, suggests a Bristol University study" (BBC, http://www.bbc.co.uk/news/health-27236297). (2) Murray, Kiryluk and Swan (1985) run a cohort of about 6000 adolescents to study the smoking behaviour and attitudes between parents and children, they find that "Children are influenced by the behaviour and attitudes of adults, especially their parents".

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we assume that during the decision making process, the reasoning process is consistent over time and the current choice is rationally consistent with the future choices. This means that each generation makes a best response to the actions of other generations and in the intertemporal NE the best responses are mutually consistent.

### 4.3.2 Censored altruism/Warm glow/Joy of giving

We begin by presenting censored/warm glow model, in which the parent (M) has desires to give because of "warm glow" or joy or social expectations. We define the "warm glow" as some extra private utility benefits to the givers. Consider a middle aged generation (M) who feels happy to invest in the dependent child's (Y) education and is censored altruistic towards the dependent Y. The Y shares consumption with the M and has no decision power. Meanwhile, the M is censored altruistic towards his/her parent (O), in the sense that the M transfers some wealth (free gift) to the O. In this censored/warm glow type altruism, the reason for giving a gift is the benefit it gives to the donor rather than to the recipient. This feature generates the fact that the choice of the gift depends on the income of the M but is independent of the preference of the O and of the dependent Y. The altruistic M allocates resources between three generations at each time period. As we assume that income is a monotonic increasing function of the human capital investment, then the higher educational investment invested by the past M at \( t - 1 \), the higher the income can be generated at \( t \) for the current M generation.

#### Decentralised economy

**Individual behavior** Each generation M at each time \( t \) has a general lifetime utility \( U_{t-1} \) as below,

\[
U_{t-1} = u^m(C^m_{t,t-1}) + (1 + \delta)^{-1}u^o(C^o_{t+1,t-1}) + \left[ \alpha u^o(C^o_{t,t-2}) + \beta v^y(f(T_{t,t})) \right]
\]

which is time additive in private utilities and also in externality (altruism), where \( C^m_{t,t-1}, C^o_{t,t-2}, C^o_{t+1,t-1} \) stand for the consumptions at time \( t \) and \( t + 1 \), for the generation born at time \( t - 1 \) and \( t - 2 \); the dependent Y and the altruistic M share a common consumption within the same household (e.g. at \( t \), the common consumption is noted as \( C^m_{t,t-1} \)); \( \alpha \) and \( \beta \) are the degrees of altruism from the M to the Y and from the M to the O; each middle aged person of any generation is assumed to have an identical degree and form of altruism towards the young and the old alive at that date; \( \delta \) is the discount rate; \( v^o(C^o_{t,t-2}) \) and \( v^y(f(T_{t,t})) \) are utility gains from being altruistic towards the O and the Y; \( f(T_{t,t}) \) is the income level which is a function of the educational investment \( T_{t,t} \).

Each M generation maximises \( U_{t-1} \) with respect to saving \( (S_{t,t-1}) \), free gift \( (R_{t,t-1}) \) to the O and the educational investment \( (T_{t,t}) \) on the Y by facing the following budget constraints,
where in period $t$, the M divides the realized income ($f(T_{t-1}, t-1)$) between consumption ($C_{t,t-1}^m$), a free gift ($R_{t,t-1}$), an educational investment ($T_{t,t}$) to its successor and savings ($S_{t,t-1}$) kept in a pot with a zero interest rate for himself; in particular, $S_{t,t-1} > 0$, we do not allow borrowing and saving is bounded by resource constraint (more details will be explained below); in period $t + 1$, the M turns to be the O who gets a free gift ($R_{t+1,t}$) from his/her successor and also the money from self-saving ($S_{t,t-1}$).

The preference for the O at $t$

The preference for the O is $U_{t-2} = u^o(C_{t,t-2})$ and his/her consumption $C_{t,t-2}$ satisfies,

$$R_{t,t-1} + S_{t-1,t-2} \geq C_{t,t-2}^o \tag{84}$$

where $U_{t-2}$ is the lifetime utility of the O; $R_{t,t-1}$ is a free gift sent by M born at $t-1$; $S_{t-1,t-2}$ comes from self-saving at $t-1$ with zero interest rate; $S_{t-1,t-2}$ is the money kept in a pot at $t-1$ by himself/herself.

The decision problem for the M at $t$

The choices ($R_{t,t-1}, S_{t,t-1}, T_{t,t}$) are made through the following maximisation process,

$$\begin{align*}
\text{obj}_{t-1} &= U_{t-1} = \max_{R_{t,t-1} \geq 0, S_{t,t-1} \geq 0, T_{t,t} \geq 0} \{[u^m(C_{t,t-1}^m) + (1 + \delta)^{-1} u^o(C_{t+1,t-1})] \tag{85} \\
&\quad + [\alpha v(C_{t,t-2}^o) + \beta v(f(T_{t,t}))]) \} \\
\text{s.t.} \quad \left\{ \begin{array}{l}
 f(T_{t-1,t-1}) - R_{t,t-1} - T_{t,t} - S_{t,t-1} \geq C_{t,t-1}^m \\
 R_{t+1,t} + S_{t,t-1} \geq C_{t+1,t-1}^o \\
 R_{t,t-1} + S_{t-1,t-2} \geq C_{t,t-2}^o 
\end{array} \right. 
\end{align*}$$

At this point, it is useful to make some standard assumptions below to guarantee the existence of a private optimum for M’s decision problems and to ensure interior solutions.

Assumption 1 (1) $u, v, f$ are differentiable (hence continuous), monotonically increasing, strictly concave; (2) $C^m = \{C_{t,t-1} | C_{t,t-1} \geq 0, t \geq 2\}$, $C^o = \{C_{t,t-2} | C_{t,t-2} \geq 0, t \geq 3\}$, $T = \{T_{t,t} | T_{t,t} > 0, t \geq 1\}$, $R = \{R_{t,t-1} | R_{t,t-1} \geq 0, t \geq 2\}$, $S = \{S_{t,t-1} | S_{t,t-1} \geq 0, t \geq 2\}$; (3) $u, v, f$ satisfy $\lim_{t \to -\infty} u'(C_{t,t-1}^m) = 0$, $\lim_{t \to -\infty} u'(C_{t,t-2}^o) = 0$, $\lim_{t \to -\infty} u'(C_{t+1,t-1}^o) = +\infty$, $\lim_{t \to -\infty} f'(T_{t,t}) = +\infty$, $\lim_{T \to -\infty} f'(T) = +\infty$, $\lim f_{T \to -\infty}$ is finite; $f(T_{1,1}) > 0$; (4) $f(0) = 0$, $T_{t,t}$ is finite and $f(T_{t,t})$ is finite.

From this, assumption 1, implies that the utilities are continuous and the feasible sets are closed and bounded.

In assumption 1, we assume that $f(T_{1,1}) > 0$, $f(0) = 0$, $T_{1,1}$ is finite and so $f(T_{1,1})$ is finite. The first middle aged
generation faces the constraint: \( f(T_{1,1}) \geq C_{2,1} + T_{2,2} + S_{2,1} > 0 \). \( S_{2,1} \) cannot be negative because generation 1 cannot borrow from the future as generation 1 is the only working generation and no one else is producing resource at the same time. From the Inada condition on utility, \( C_{2,1} \) must be non-negative, then \( T_{2,2} \) must be finite. This implies that \( f(T_{2,2}) \) is finite and so next period since \( S_{3,2} + R_{3,2} \) is bounded below by the saving of the current old \( (S_{3,1}, \) this is the most that the current old can lend to the current \( M \). \( T_{3,3} \) must be finite and hence the resources available to the middle aged born at \( t = 2 \) have only finite resources to allocate. This means that \( T_{4,4} \) must be finite. Continuing in this way, at every \( t, T_{t,t} \) is finite. Hence the feasible set is bounded at every \( t \). Closedness of the feasible set at \( t \) is ensured by the weak inequalities defining the feasibility condition. Since the feasible set is closed and bounded, and utility is continuous, an optimum exists.

Part (3) of assumption 1 ensures generally the solution is interior since if training of the young or consumption of any generation is zero, there is an infinite marginal utility gain to be had from a marginal increase in the zero valued variable. The concavity and convexity conditions on the payoff functions and the training function also ensure the optimum will be unique. In addition, we assume that goods are storable and there is no natural growth rate intertemporally \( (e.g. S_{t,t}, 1 \text{ grows with zero interest rate}) \).

Substituting out the constraints, we derive the first order conditions for \( R_{t,t-1}, S_{t,t-1} \) and \( T_{t,t} \) from \( Obj_{t,t-1} \)

\[
Obj_{t,t-1} = \max_{R_{t,t-1} \geq 0, x_{t,t-1} \geq 0} \left\{ [u^m(f(T_{t-1,t-1}) - R_{t,t-1} - T_{t,t} - S_{t,t-1}) + (1 + \delta)^{-1} u^o(R_{t+1,t} + S_{t,t-1})] + [\alpha v(R_{t,t-1} + S_{t-1,t-2}) + \beta v(f(T_{t,t}))]\right\} (86)
\]

The Kuhn Tucker conditions are

\[
\frac{\partial Obj_{t,t-1}}{\partial R_{t,t-1}} = 0, T_{t,t} \frac{\partial Obj_{t,t-1}}{\partial T_{t,t}} = 0, S_{t,t-1} \frac{\partial Obj_{t,t-1}}{\partial S_{t,t-1}} = 0 (87)
\]

In fact we know that \( T_{t,t} > 0 \) so

\[
Foc, R_{t,t-1} : \ (1) \text{ if } R_{t,t-1} > 0, \text{ then } u_{C_{t-1,t}}^m = \alpha v_{C_{t-1,t}}^o \]
\[
Foc, R_{t,t-1} : \ (2) \text{ if } R_{t,t-1} = 0, \text{ then } u_{C_{t-1,t}}^m > \alpha v_{C_{t-1,t}}^o \]
\[
Foc, T_{t,t} : \ (1) \text{ if } T_{t,t} > 0, \text{ then } u_{C_{t-1,t}}^m = \beta v_{f_{T_{t,t}}} \]
\[
Foc, S_{t,t-1} : \ (1) \text{ if } S_{t,t-1} > 0, \text{ then } u_{C_{t-1,t}}^m = (1 + \delta)^{-1} u_{C_{t+1,t-1}}^o \]
\[
Foc, S_{t,t-1} : \ (2) \text{ if } S_{t,t-1} = 0, \text{ then } u_{C_{t-1,t}}^m > (1 + \delta)^{-1} u_{C_{t+1,t-1}}^o \]

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where the corner solutions occur when the marginal costs are higher than the marginal benefits \((u_{Cm_{t-1}}' > \alpha v_{C_{t-2}}' ; u_{Cm_{t-1}}' > \beta v_{f T_{t-1}}' ; u_{C_{t-1}}' > (1 + \delta)^{-1} u_{C_{t+1,t-1}}')\). Therefore, individuals have no motivations to make any \(R_{t,t}, S_{t,t} \) and \(T_{t,t}\).

Here if we only consider interior solutions, \(R_{t,t} > 0, S_{t,t} > 0, T_{t,t} > 0\), (the Inada condition guarantees this for \(T_{t,t}\)), then we get

\[
u_{Cm_{t-1}}' = \alpha v_{C_{t-2}}' = \beta v_{f T_{t-1}}' = (1 + \delta)^{-1} u_{C_{t+1,t-1}}'
\]

which imply that the optimal value for \(R_{t,t-1}\) is obtained when the marginal cost of giving away one unit of \(C_{m_{t-1}}\) equals the marginal benefit of being altruistic \((\alpha v_{C_{t-2}}')\); the optimal saving is chosen when the marginal rate of substitution \((\frac{u_{Cm_{t-1}}'}{u_{C_{t+1,t-1}}'})\) equals to the discount factor \((1 + \delta)^{-1}\); consumptions between \(t\) and \(t+1\) are smoothed when \(\delta = 0\); the optimal \(T_{t,t}\) is chosen when the ratio of marginal cost and benefit \((\frac{u_{Cm_{t-1}}'}{\beta e_{t}})\) equals to the marginal productivity of educational investment (e.g. market wage rate).

**Private equilibrium**  In this OLG framework with censored/warm glow type altruism in a non-cooperative Nash environment, in each period, the middle aged generation makes the choices knowing the actions made by the O in the past and the future M will be fully committed. The current M is assumed to have perfect foresight on future generation’s free gift choice. The altruistic M makes choices corresponding to the choices made by the past M and the future M.

**Reaction curves of choice variables**

Except from the fact that variables are time-varying, in each time period we have three identical agents (O, M and Y) and each agent has time-consistent preferences over time. Knowing this, the choice variables in different time periods will have the same properties but with an updated time subscript. All the choice variables before \(t^*\) will be functions of the initial \(T_{t,1}\) and the free gift at \(t^* + 1\) \((R_{t^*+1,t^*})\) and some exogenous parameters \((\alpha, \beta, \delta)\).

In an infinite time horizon setting, if we choose a sub interval (Fig 42) such as time \(t^*\) to \(t^* + s\), then given that all the choice variables before \(t^*\) and after \(t^* + s\) are held at their full intertemporal NE values, within the sub interval, the choices must be best responses to choices outside of the sub interval and choices at other period within the sub interval. So we can find necessary conditions for an intertemporal NE by looking at mutual best responses within arbitrary sub intervals. This version of Bellmans Principle of Optimality applies also to the subsequent analysis of an inertemporal social optimum.

The reaction curves for interior \(R_{t,t-1}, S_{t,t-1}\) and \(T_{t,t}\) for generation born at \(t-1\) are
Figure 42: Sub-period NE

\[
R_{t,t-1} = R_{t+1,t-1}^{RC} (\alpha, \beta, \delta, T_{t-1,t-1}, S_{t-1t-2}, R_{t+1,t}) \tag{90}
\]
\[
S_{t,t-1} = S_{t+1,t-1}^{RC} (\alpha, \beta, \delta, T_{t-1,t-1}, S_{t-1t-2}, R_{t+1,t})
\]
\[
T_{t,t} = T_{t+1t+1}^{RC} (\alpha, \beta, \delta, T_{t-1,t-1}, S_{t-1t-2}, R_{t+1,t+1})
\]

where \(T_{t-1,t-1}, S_{t-1t-2}\) are assumed to be fixed and are set at NE level already.

The reaction curves for interior \(R_{t+1,t}, S_{t+1,t}\) and \(T_{t+1t+1}\) for generation born at \(t\) are as,

\[
R_{t+1,t} = R_{t+1,t}^{RC} (\alpha, \beta, \delta, T_{tt}, S_{tt-1}, R_{t+2,t+1}) \tag{91}
\]
\[
S_{t+1,t} = S_{t+1,t}^{RC} (\alpha, \beta, \delta, T_{tt}, S_{tt-1}, R_{t+2,t+1})
\]
\[
T_{t+1t+1} = T_{t+1t+1}^{RC} (\alpha, \beta, \delta, T_{tt}, S_{tt-1}, R_{t+2,t+1})
\]

where \(R_{t+2,t+1}\) is assumed to be fixed and set at NE level already.

So the intertemporal NE for generations \(t - 1\) and \(t\) at time \(t\) and \(t + 1\) can be expressed as,

\[
R_{t,t-1} = R_{t,t-1}^{NEW} (\alpha, \beta, \delta, T_{t-1,t-1}, S_{t-1t-2}, R_{t+2,t+1}) \tag{92}
\]
\[
S_{t,t-1} = S_{t,t-1}^{NEW} (\alpha, \beta, \delta, T_{t-1,t-1}, S_{t-1t-2}, R_{t+2,t+1})
\]
\[
T_{tt} = T_{tt}^{NEW} (\alpha, \beta, \delta, T_{t-1,t-1}, S_{t-1t-2}, R_{t+2,t+1})
\]
\[
R_{t+1,t} = R_{t+1,t}^{NEW} (\alpha, \beta, \delta, T_{t-1,t-1}, S_{t-1t-2}, R_{t+2,t+1})
\]
\[
S_{t+1,t} = S_{t+1,t}^{NEW} (\alpha, \beta, \delta, T_{t-1,t-1}, R_{t+2,t+1})
\]
\[
T_{t+1t+1} = T_{t+1t+1}^{NEW} (\alpha, \beta, \delta, T_{t-1,t-1}, S_{t-1t-2}, R_{t+2,t+1})
\]

where all the choice variables \((T_{t-1,t-1}, S_{t-1t-2}, R_{t+2,t+1})\) outside of periods \(t\) and \(t + 1\) for other generations (not generations \(t - 1\) and \(t\)) are assumed to be fixed and are set at NE level already; the intertemporal choice variables
are functions of the training and saving choices made by the past M and the free gift sent by the future M. This feature generates intergenerational Nash Equilibrium and introduces the dynamic process of choices below.

**The dynamics of choices**

Correspondingly, the dynamic process of choice variables can be obtained below. The past HC investment, savings and future free gift choices accumulate into the current $T_{t,t}$. The dynamic process of $T_{tt}$ is,

$$T_{t,t} = T_{t,t}(\alpha, \beta, \delta, T_{t-1,t-1}, S_{t-1,t-2}, R_{t+2,t+1}) \quad (93)$$

where $T_{t,t}$ has backward looking ($T_{t-1,t-1}, S_{t-1,t-2}$) and forward looking ($R_{t+2,t+1}$) elements/determinants. The current HC investment choice is influenced by past educational investment and saving choices and also by future repayments (gift sent by the future generation).

Suppose the initial HC investment is $T_{1,1}$, then the subsequent choices can be obtained as,

**time = 2**

$$S_{2,1} = S(\alpha, \beta, \delta, T_{1,1}, R_{3,2}) \quad (94)$$

$$T_{2,2} = T(\alpha, \beta, \delta, S_{2,1}, R_{3,2})$$

$$= T(\alpha, \beta, \delta, T_{1,1}, R_{3,2})$$

**time = 3**

$$R_{3,2} = R(\alpha, \beta, \delta, S_{2,1}, T_{2,2}, R_{4,3}) \quad (95)$$

$$= R(\alpha, \beta, \delta, T_{1,1}, R_{4,3})$$

$$S_{3,2} = S(\alpha, \beta, \delta, T_{2,2}, S_{2,1}, R_{4,3})$$

$$= S(\alpha, \beta, \delta, T_{1,1}, R_{3,2}, R_{4,3})$$

$$= S(\alpha, \beta, \delta, T_{1,1}, R_{4,3})$$

$$T_{3,3} = T(\alpha, \beta, \delta, T_{2,2}, S_{2,1}, R_{4,3})$$

$$= T(\alpha, \beta, \delta, T_{1,1}, R_{4,3})$$

**time = 3**

$$R_{4,3} = R(\alpha, \beta, \delta, T_{1,1}, R_{5,4}) \quad (96)$$

$$S_{4,3} = S(\alpha, \beta, \delta, T_{1,1}, R_{5,4})$$

$$T_{4,4} = T(\alpha, \beta, \delta, T_{1,1}, R_{5,4})$$
\[
S_{t,t-1} = S^{NE}(\alpha, \beta, \delta, T_{1,1}, R_{t+1,t}) \quad \text{\quad (97)}
\]
\[
R_{t,t-1} = R^{NE}(\alpha, \beta, \delta, T_{1,1}, R_{t+1,t})
\]
\[
T_{t,t} = T^{NE}(\alpha, \beta, \delta, T_{1,1}, R_{t+1,t})
\]

Overall, the choice variables at \( t \) are determined by the initial training \( T_{1,1} \) and future free gift \( R_{t+1,t} \) sent by the future generation \( M \), who is assumed to be fully committed. And the current generation has perfect foresight on choice \( R_{t+1,t} \).

**Welfare Optimum**  
In this section, similarly to but also differently from the welfare optimum idea\(^{58}\) in Bernheim’s (1989) paper, we study whether the optimal condition for choice variables in the private NE can also be an optimum in a social system. The idea is that for an interior solution, given all the choice variables and resources outside periods \( t \) and \( t+1 \) for generations \( t-1 \) and \( t \) are fixed and set at NE level, if the social planner reallocates resources between periods \( t \) and \( t+1 \) for the \( t^* - 1 \) generation and the \( t^* \) generation, then what are the impacts of the reallocations on the social welfare function? Can the social planner improve social welfare using the private NE conditions by any other resource reallocations? To answer these questions, we study the changes in lifetime utility in social welfare function for generations \( t^* - 1 \) and \( t^* \) using private first order conditions for choice variables at \( t^* \) and \( t^* + 1 \). We define \( \psi \) as a social welfare function, which assumed to be additively separable and linear,

\[
\psi(U_1, U_2, \ldots U_{t-1} \ldots)
\]

\[
= \lambda_1[U_1] + \sum_{t=3}^{\infty} \lambda_{t-1}[U_{t-1}]
\]

\[
= \{ \lambda_1[u^m(C^m_{2,1})] + (1 + \delta)^{-1}u^o(C^o_{3,1}) + \beta v(f(T_{2,2})) \}
\]

\[
+ \sum_{t=3}^{\infty} \lambda_{t-1}[u^m(C^m_{t,t-1}) + (1 + \delta)^{-1}u^o(C^o_{t+1,t-1}) + \alpha v(C^o_{t,t-2}) + \beta v(f(T_{t,t}))]
\]

where \( U_j, j = \{1, 2, \ldots\} \), is lifetime utility of each generation; where \( \sum_{t=2}^{\infty} \lambda_{t-1} = 1 \); the society starts from one \( Y \) generation who has no parent; the initial generation does not need to care about the consumption of the non-existent

\(^{58}\)He considers a transfer of consumption, \( \alpha \), from generation \( t^* - 1 \) to \( t^* \). He then looks at the effect of this transfer on welfare by using dynastic equilibrium \((\max_{C^o_{t}, C^m_{t}} \sum_{t=0}^{\infty} \delta^t v(C^o_{t}, C^m_{t})|b_{t-1} + w_{t} = C^o_{t} + \beta^{-1}(C^o_{t} + b_{t}))\) (not private NE) conditions to see whether the welfare optimum can be achieved. In Bernheim’s dynamic equilibrium idea, the initial generation acts as a dictator who maximises lifetime utilities including infinite altruistic links towards all the future generations. But our private NE applies to any generation who maximises lifetime utilities including altruistic links.

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O so that the format of $U_1$ is different from others ($U_k$, $k = \{1, 2, \ldots\}$); from $t \geq 3$, the social planner recognizes there are three generations living within the same period; the planner knows that the altruistic M shows altruism towards the O and the future M (who is Y at this moment).

Here, we also only consider interior choices. And goods are storable, the social planner can move goods intertemporally; the planner make choices on $S_t \geq 0$ to decide the amount of goods to store at time $t$.

At each period, the planner faces the aggregate resource constraint below

$$t = 2 : C_{2,1} + C_{3,1} + T_{2,2} + S_{2,1} = f(T_{1,1})$$

$$t \geq 3 : C_{t,t-1}^m + C_{t,t-2}^0 + T_{tt} + S_t = f(T_{t-1t-1}) + S_{t-1}$$

where aggregate resource is completely allocated in each period.

Suppose the planner transfers some resources within time $t$ and $t + 1$ for generations $t^* - 1$ and $t^*$ but the aggregate resources outside those two periods are held at their NE values. The planner only needs to consider the aggregate resource for those two periods below,

$$\text{time } t : C_{t,t-1}^m + C_{t,t-2}^0 + T_{tt} + S_t = f(T_{t-1t-1}) + S_{t-1}$$

$$\text{time } t + 1 : C_{t+1,t}^m + C_{t,t-1}^0 + T_{t+1t+1} = f(T_{tt}) + S_t$$

where the choices (e.g. $C_{t,t-2}^0, T_{t-1t-1}, S_{t-1}, T_{t+1t+1}$) outside periods $t$ and $t + 1$ are fixed at NE level.

The planner reallocates some units of resources at time $t$ given the total resources at $t$ is fixed ($f(T_{t-1t-1}) + S_{t-1}$). The reallocations must satisfy

$$d(C_{t,t-1}^m) + d(S_t) + d(T_{tt}) = 0$$

And in the next time period, the social planner reallocates the resources at $t + 1$. The reallocation at $t + 1$ satisfies,

$$dC_{t+1,t} + dC_{t+1,t-1} = f'(T_{t+1})dT_{t+1} + dS_t$$

Besides all those choices set at NE level, the social planner reallocates the $C_{t,t-1}^m, T_{tt}, S_t, C_{t+1,t}^m, C_{t,t-1}^0$. It is worth noticing that there are intertemporal effects ($f'(T_{t,t})dT_{t,t}$) when $d(T_{tt}) \neq 0$. This is one of the reasons why we study intertemporal NE within two adjacent generations and periods.

What effect does this have on welfare? Differentiating,
\[ d\psi(U_1, U_2, ... U_{t-1}...) = d\psi(\overline{U}_1, \overline{U}_2, ... \overline{U}_{t-2}, U_{t-1}^*, U_t^*, \overline{U}_{t+1}^*) \]  
\[ = d\lambda_{t-1}U_{t-1} + d\lambda_tU_t^* \]  

(103)

where \( \overline{U}_j, j = \{1, 2, \ldots \}, j \neq i \), is set at NE level. For example, the assumption for fixed choices \( (C_{t,t-2}^o, T_{t-1,t-1}, S_{t-1}) \) ensures \( \overline{U}_{t-2} \) is fixed at NE level, where \( \overline{U}_{t-2} = u^m(C_{t,t-1,t-2}) + (1 + \delta)^{-1}u^o(C_{t,t-2}^o) + \alpha u^o(C_{t-1,t-3}^o) + \beta v(f(T_{t-1,t-1})) \); the aggregate resource at \( t - 1 \) is \( C_{t-1,t-2}^o + C_{t-1, t-3}^o + T_{t-1,t-1} + S_{t-1} = f(T_{t-2,t-2}) + S_{t-2} \) set at NE level.

\[ d\lambda_{t-1}U_{t-1} = \lambda_{t-1}[u^m_{C_{t-1,t-1}, dC_{t,t-1}^m} + (1 + \delta)^{-1}u^o_{C_{t+1,t-1}, dC_{t+1,t-1}^o} \]
\[ + \beta v' f' dT_{t,t} + \alpha v'_{C_{t-1,t-2}, dC_{t,t-2}^o}] \]

(104)

where, \( dC_{t,t-2}^o = 0 \)

\[ d\lambda_tU_t^* = \lambda_t[u^m_{C_{t+1,t}, dC_{t+1, t}^m} + (1 + \delta)^{-1}u^o_{C_{t+2,t}, dC_{t+2,t}^o} \]
\[ + \beta v' f' dT_{t+1,t+1} + \alpha v'_{C_{t+1,t-2}, dC_{t+1,t-1}^o}] \]

(105)

where, \( dT_{t+1,t+1} = 0, dC_{t+2,t} = 0 \)

here, \( dC_{t,t-2}^o = 0, dT_{t+1,t+1} = 0, dC_{t+2,t} = 0 \), the choice variables \( C_{t,t-2}^o \), \( C_{t+2,t}^o \) and \( T_{t+1,t+1} \) are set at NE level already and are not influenced by the reallocation.

Given the foci in the decentralised NE, \( u^o_{C_{t+1,t-1}} = \beta v' f'T_{t,t} = (1 + \delta)^{-1}u^o_{C_{t+1,t-1}, dC_{t+1,t-1}^o} \), \( u^o_{C_{t+1,t-1}} = \alpha v'_{C_{t+1,t-1}} \), we get

\[ d\lambda_{t-1}U_{t-1} = \lambda_{t-1}[u^m_{C_{t-1,t-1}, dC_{t,t-1}^m} + dC_{t+1,t-1}^o + dT_{t,t}] \]
\[ d\lambda_tU_t^* = \lambda_t[u^m_{C_{t+1,t}, dC_{t+1, t}^m} + dC_{t+1,t}^o] \]

(106)

then

\[ d\lambda_{t-1}U_{t-1} + d\lambda_tU_t^* \]
\[ = \lambda_{t-1}[u^m_{C_{t-1,t-1}, dC_{t,t-1}^m} + dC_{t+1,t-1}^o + dT_{t,t}] \]
\[ + \lambda_t[u^m_{C_{t+1,t}, dC_{t+1, t}^m} + dC_{t+1,t}^o] \]

(107)

Given (101)& (102), we get

\[ = \lambda_{t-1}u^m_{C_{t-1,t-1}, [-d(S_t) + dC_{t+1,t-1}]} \]
\[ + \lambda_t[u^m_{C_{t+1,t}}[f'(T_{t,t})dT_{tt} + dS_t]] \]

(108)
We assume that the planner allocates some units of resource from $C_{t,t-1}$ to $T_{t,t}$ and $S_{t-1}$, then $dC_{t,t-1} < 0$, $dT_{t,t} > 0$, $dS_{t} > 0$; $\Rightarrow [f'(T_{t,t})dT_{t,t} + dS_{t}] > 0$.

As soon as $-d(S_{t}) + dC_{t+1,t-1} \geq 0$ (if the planner removes more resources intertemporally to the O’s consumption, then planner has less resource to store intertemporally, $dC_{t+1,t-1} \geq d(S_{t})$), $\lambda_{t-1} \neq 0, \lambda_{t} \neq 0$, then

$$d\lambda_{t-1}U_{t-1} + d\lambda_{t}U_{t} > 0$$

This shows that the planner can improve on the intertemporal NE and that the social optimum cannot be obtained through private NE conditions.

**Proposition** This shows that the planner can improve the intertemporal NE and that suppose that $\lambda_{t-1} \geq 0$, for $t \geq 2$, equations (101)& (102) hold, the optimal conditions for consumptions, trainings and savings in the private NE with censored altruism cannot be a social optimum.

### 4.3.3 Some possible corrections or policy suggestions

**Pure altruism/love** Above, we learned that the private NE with censored altruism cannot achieve a social optimum. In this section, we suggest a few possible corrections or policy suggestions to provide some instruments or different type of altruism to obtain a social optimum in the intertemporal NE. First, we turn to the pure altruism/love to see whether the private Nash equilibrium in this type of altruism can be a social optimum. A substantial body of research has established that a variety of important issues hinge upon the nature of pure altruism. In particular, Bernheim (1989) builds a framework with parents who are altruistically linked to all future children through bequests in an infinite time horizon to explore the welfare properties. He finds that welfare optima cannot be obtained through this type of altruism. Following the same route, we develop a slightly different framework from Bernheim’s work to investigate the welfare properties and private NE. In Bernheim’s framework, the intrahousehold transfer is just one way (from senior to junior members), but we link parents with all future children through two ways, intrahousehold transfers. The two ways are upstream and downstream flows. Each generation altruistically contributes to the future generation but also receives financial/cash flows in the next time period. This setting makes individuals not only enjoy the altruistic benefit but also pay for the cost in the next round. Through this, the role of being altruistic gets reversed among lifetime periods for each individual. Therefore, different from Bernheim’s model, having reciprocal altruism might generate different welfare properties. We develop a type of pure altruism with two way transfer and study the properties of this type of pure altruism. In order to give a neat explanation, we will follow the same notations (e.g. $C_{t,t-1}$ means the consumption of the middle aged generation at $t$ with birth date $t-1$) and assumptions (e.g. all the utilities are continuous, monotonic increasing
in its arguments, strictly concave, constraints are closed and bounded, the sum of welfare function is bounded as those in the censored altruism. The preferences of this type of altruism are different and are listed below.

Decentralised economy Private equilibrium conditions

The decision problem for the \( M \) at \( t \)

The choices \( (R_{t,t-1}, S_{t,t-1}, T_{t,t}) \) are made through the following maximisation process,

\[
\text{obj}_{t,t-1} = \max_{M's \ lifetime \ utilities} \left\{ \begin{array}{l} 
\frac{U_{t-1}}{} = R_{t,t-1} \geq 0, S_{t,t-1} \geq 0, T_{t,t} \geq 0 \left\{ u^m(C_{t,t-1}^m) + (1 + \delta)^{-1} u^a(C_{t+1,t-1}^a) \right\} \\
\text{altruism \ towards} \ O's \ current \ utility \\
+ \left\{ \begin{array}{l} \alpha u^a(C_{t,t-2}) \\
\text{altruism \ towards} \ Y's \ lifetime \ utilities \\
\beta U_{t+1} \end{array} \right\} \\
\text{s.t.} \left\{ \begin{array}{l} f(T_{t-1,t-1}) - R_{t,t-1} - T_{t,t} - S_{t,t-1} \geq C_{t,t-1}^m \\
R_{t+1,t} + S_{t,t-1} \geq C_{t+1,t-1}^m \\
f(T_{t,t}) - R_{t+1,t} - T_{t+1,t+1} - S_{t+1,t} \geq C_{t+1,t}^m \\
R_{t+2,t+1} + S_{t+1,t} \geq C_{t+2,t}^m \\
\end{array} \right\} 
\right\}
\]

where

\[
U_t = u^m(C_{t+1,t}^m) + (1 + \delta)^{-1} u^a(C_{t+2,t}^a) + \alpha u^a(C_{t+1,t-1}^a) + \beta U_{t+1}
\]

where the altruistic \( M \) cares about the last period utility of the current \( O \) who is at his/her last time period so \( u^a(C_{t,t-2}) \) enters into \( M's \ U_{t-1} \) but the \( M \) cares about the lifetime utilities of the \( Y \) who also cares about the lifetime utility of the future \( Y \). So actually, the current \( M \) interacts with all future \( Y \) in an infinite time horizon; the resource constraints are as the same as those in the censored/warm glow type of altruism.

The objective function after substituting out the constraints becomes

\[
\text{obj}_{t,t-1} = \max_{R_{t,t-1} \geq 0, S_{t,t-1} \geq 0, T_{t,t} \geq 0} \left\{ \begin{array}{l} u^m(f(T_{t-1,t-1}) - R_{t,t-1} - T_{t,t} - S_{t,t-1}) \\
+(1 + \delta)^{-1} u^a(R_{t+1,t} + S_{t,t-1}) \\
+\beta u^a(f(T_{t,t}) - R_{t+1,t} - T_{t+1,t+1} - S_{t+1,t}) \\
+(1 + \delta)^{-1} u^a(R_{t+2,t+1} + S_{t+1,t}) + \alpha u^a(R_{t+1,t} + S_{t,t-1} + \beta U_{t+1}) \end{array} \right\} 
\]

\[
\text{obj}_{t,t-1} = \max_{R_{t,t-1} \geq 0, S_{t,t-1} \geq 0, T_{t,t} \geq 0} \left\{ \begin{array}{l} u^m(f(T_{t-1,t-1}) - R_{t,t-1} - T_{t,t} - S_{t,t-1}) \\
+(1 + \delta)^{-1} + \alpha \beta u^a(R_{t+1,t} + S_{t,t-1}) \\
+\alpha u^a(R_{t+1,t} + S_{t,t-1} - 2) \\
+\beta u^a(f(T_{t,t}) - R_{t+1,t} - T_{t+1,t+1} - S_{t+1,t}) \\
+(1 + \delta)^{-1} \beta u^a(R_{t+2,t+1} + S_{t+1,t} + \beta^2 U_{t+1}) \end{array} \right\} 
\]

\[\text{117}\]
where

\[ U_{t+1} = u^m(f(T_{t+1,t+1}) - R_{t+1,t+1} - T_{t+2,t+2} - S_{t+2,t+2}) \]

\[ + (1 + \delta)^{-1} u^\sigma(R_{t+3,t+2} + S_{t+2,t+1}) + \alpha u^\sigma(R_{t+2,t+1} + S_{t+1,t}) + \beta U_{t+2} \]

(114)

where all the variables follow the same notations as in the censored altruism.

The private optimum exists following assumption 1. Following our earlier argument for a finite time horizon, starting from a finite initial level of training, subsequent levels of feasible training are finite. Hence, the feasible sets are bounded and closed. Continuity of the utility function requires some care because of the infinite horizon of the pure altruism. From (112), we can write utility for middle aged at \( t \) as

\[ obj_{t-1} = \Sigma_{s=t}^\infty [u^m(C_{t,s-1})\beta^s + (1 + \delta)^{-1} u^\sigma(C_{t,s-2})\beta^s] \]

(115)

Since consumption is bounded above, \( C_{t,s-1} \) and \( C_{t,s-2} \) are both bounded above and so are their corresponding utilities so that

\[ u^m(C_{t,s-1}) \leq \overline{u}^m, u^\sigma(C_{t,s-2}) \leq \overline{u}^\sigma \]

Thus

\[ obj_{t-1} \leq (\overline{u}^m + \overline{u}^\sigma)\Sigma_{s=t}^\infty \beta^s \]

If \( \Sigma_{s=t}^\infty \beta^s \) is finite then \( obj_{t-1} \) is bounded above and even as \( t \to \infty \), the utility function is continuous. So there must be an optimal solution, again from the concavity/convexity, it is unique. With the Inada conditions holding for each utility and training function, we can take it to be interior. The Kuhn Tucker conditions are

\[ R_{t,s-1} \frac{\partial obj_{t-1}}{\partial R_{t,s-1}} = 0, T_{t,s} \frac{\partial obj_{t-1}}{\partial T_{t,s}} = 0, S_{t,s-1} \frac{\partial obj_{t-1}}{\partial S_{t,s-1}} = 0 \]

(116)

The focs for interior solutions are,

\[ Foc, R_{t,s-1} : u'_{C^o_{t,s-2}} = \alpha u_C'_{t,s-2} \]

(117)

\[ Foc, T_{t,s} : u'_{C^m_{t+1,s}} = \beta u_C'_{t+1,s} f^*_T_{t,s} \]

\[ Foc, S_{t,s-1} : u'_{C^m_{t,s-1}} = [(1 + \delta)^{-1} + \alpha \beta] u_C'_{t+1,s-1} \]

Then for the interior solutions we get

\[ u'_{C^m_{t,s-1}} = \alpha u_C'_{t,s-2} = \beta u_C'_{t+1,s} f^*_T_{t,s} = [(1 + \delta)^{-1} + \alpha \beta] u_C'_{t+1,s-1} \]

(118)
where the optimal $R_{t,t-1}$ is chosen when the cost of giving up one unit of $C^m_{t,t-1}$ is compensated by the benefit $\alpha u^{'C_{t-2}}$; the optimal training is chosen when the marginal rate of substitution equals the product of the degree of altruism and the marginal productivity of training investment ($\frac{u^{C_{t+1,t}}}{u^{C_{t+1,t}}_t} = \beta f^{'}_{T_{t,t}}$); the intertemporal consumption is smoothed between $t$ and $t+1$ for generation $t-1$ when $\delta, \alpha, \beta = 0$.

By updating time subscripts, we get

$$u^{C_{t+1,t}} = \alpha u^{C_{t+1,t}}_t = \beta u^{C_{t+2,t+1}} f^{'}_{T_{t+1,t+1}} = [(1 + \delta)^{-1} + \alpha \beta] u^{C_{t+2,t}}$$ \hspace{1cm} (119)

Interestingly through equations (118) and (119), we get

$$f^{'}_{T_{t,t}} = \frac{[(1 + \delta)^{-1} + \alpha \beta]}{\beta \alpha} > 1$$ \hspace{1cm} (120)

where the marginal productivity of educational investment is constant and is bigger than 1; one unit of $T_{t,t}$ generates more than one unit of output. We will compare the private marginal productivity of $T_{t,t}$ with the social one to investigate whether individuals are under or over training in the private system.

Welfare optimum

The social welfare function is
\[ \psi(U_1, U_2, \ldots U_{t-1}) \]

\[ = \lambda_1[U_1] + \sum_{t=3}^{\infty} \lambda_{t-1}[U_{t-1}] \]

\[ = \lambda_1[u^m(C_{m,1})] + (1 + \delta)^{-1}u^o(C_{3,1}) \]

\[ + \beta[u^m(C_{3,2})] + (1 + \delta)^{-1}u^o(C_{4,2}) + \alpha u^o(C_{3,1}) \]

\[ + \beta[u^m(C_{4,3})] + (1 + \delta)^{-1}u^o(C_{5,3}) + \alpha u^o(C_{4,2}) \]

\[ + \beta[u^m(C_{5,4})] + (1 + \delta)^{-1}u^o(C_{6,4}) + \alpha u^o(C_{5,3}) \]

\[ + \beta[u^m(C_{6,5})] + (1 + \delta)^{-1}u^o(C_{7,5}) + \alpha u^o(C_{6,4}) + \beta[\ldots] \]

\[ + \lambda_2[u^m(C_{3,2})] + (1 + \delta)^{-1}u^o(C_{4,2}) + \alpha u^o(C_{3,1}) \]

\[ + \beta[u^m(C_{4,3})] + (1 + \delta)^{-1}u^o(C_{5,3}) + \alpha u^o(C_{4,2}) \]

\[ + \beta[u^m(C_{5,4})] + (1 + \delta)^{-1}u^o(C_{6,4}) + \alpha u^o(C_{5,3}) \]

\[ + \beta[u^m(C_{6,5})] + (1 + \delta)^{-1}u^o(C_{7,5}) + \alpha u^o(C_{6,4}) + \beta[\ldots] \]

\[ + \lambda_3[u^m(C_{4,3})] + (1 + \delta)^{-1}u^o(C_{5,3}) + \alpha u^o(C_{4,2}) + \]

\[ + \beta[u^m(C_{5,4})] + (1 + \delta)^{-1}u^o(C_{6,4}) + \alpha u^o(C_{5,3}) \]

\[ + \beta[u^m(C_{6,5})] + (1 + \delta)^{-1}u^o(C_{7,5}) + \alpha u^o(C_{6,4}) + \beta[\ldots] \]

\[ + \lambda_4[u^m(C_{5,4})] + (1 + \delta)^{-1}u^o(C_{6,4}) + \alpha u^o(C_{5,3}) \]

\[ + \beta[u^m(C_{6,5})] + (1 + \delta)^{-1}u^o(C_{7,5}) + \alpha u^o(C_{6,4}) + \beta[\ldots] \]

\[ + ... \]

where \( U_1 \) and \( U_{t-1} \), \( t \geq 3 \), stand for the lifetime utilities of all generations; \( \lambda_{t-1} \), \( t \geq 2 \), is just social weights set by the social planner.

Now we turn to welfare optimum. With some modifications, we can apply previous arguments to be sure that a social optimum exists. Again, the feasible sets are closed and bounded each period of time if we start with finite initial training. To avoid problems of convergence of utility at infinity, we would like to ensure that the social welfare function remains bounded. As with the private optimum, we can write (121) as

\[ \sum_{t=1}^{\infty} \lambda_{t-1}[\sum_{s=1}^{\infty} \beta^s u^m(C_{t-1,s})] \leq (\overline{m} + \overline{r}) \sum_{t=1}^{\infty} \sum_{s=1}^{\infty} \lambda_{t-1} \beta^s \]

(122)

where \( \overline{m} \) and \( \overline{r} \) are the bounds on per period utility.
If $\Sigma^\infty_t \Sigma_{s=t}^\infty \lambda_{t-1} \beta^s$ is finite, we have a welfare function which is bounded above. This is likely to occur when $\beta$ is low ($< 0.5$) and the sequence of weights of different generations $\lambda_{t-1}$ falls sufficiently fast through time.

Starting from a private NE, can a planner improve social welfare by changing any allocations? If he can, then NE cannot be socially optimum. Given the features of this pure altruism: (1) infinite caring from generation 1 to generation $\infty$; (2) feedback effects from the future generation to the current middle aged generation through reciprocal altruism, the reallocation within random periods $t$ and $t+1$ will have chain effects on lifetime utilities of all the previous generations (the future generation cares about all previous generations through reciprocal altruism). Therefore, we study the impacts of reallocations on welfare function through two cases:

1. the social planner only reallocates resource from generation 1 to 2 within time 2 and 3 since an optimum must be optimal within all sub intervals. In this case, there are no chain effects (there is no resource allocation from 1 to $\geq 3$ so generation $\geq 3$ are not affected; only generation 2 is affected).

2. the social planner reallocates resources between time $t$ and $t+1$ (since an optimum must be optimal within all sub intervals). The chain effects are considered here (generations $\leq t+1$ are all affected by the reallocation process).

Case (1)

Suppose we remove some units of $C_{m,2}$ to $T_{2,2}$ and $S_2$ ($dC_{m,2} < 0, dT_{2,2}^c > 0, dS_2 > 0$), other variables outside time $t = 2, 3$ as well as the aggregate resources in each period are kept the same, then what are the impacts of the reallocations on social welfare function? It is worth noticing that the reallocation from generation 1 to 2 has no impacts on future generations’ decisions.

Here, the social planner maximises the social welfare function subject to the budget constraints below,

$$
t = 2 : (C_{m,2} + T_{2,2}^c + S_2) = f(T_{1,1}) \tag{123}
$$

$$
t \geq 3 : (C_{m,t-1} + C_{o,t-2} + T_{t,t}^c + S_t) = f(T_{t-1,1}) + S_{t-1}
$$

Assumed above, the aggregate resources out outside time $t = 2, 3$ stay the same. The social planner only reallocates resources within $t = 2, 3$. Any allocations are feasible if they satisfy

$$
dC_{m,2} + dT_{2,2}^c + dS_2 = 0 \tag{124}
$$

$$
dC_{m,3} + dC_{3,1} = \int (T_{2,2})dT_{2,2} + dS_2
$$
where \( dC_{m1} < 0, dT_{2,2} > 0, dS_2 > 0, dT_{3,3} = 0, dS_3 = 0; \forall t \geq 4, dC_{m1} = 0, dC_{t-t-2} = 0, dT_{t-t} = 0, dS_t = 0, df(T_{t-1-t-1}) = 0. \)

Then

\[
dC_{3,2} + dC_{3,1} = f'(T_{2,2})dT_{2,2} + dS_2 > 0
\]

(125)

The impacts of the reallocations on the social welfare function are

\[
d\psi = \lambda_1u_{m1}dC_{2,1} + (1 + \delta)^{-1}u_{C_{3,3}}dC_{3,1}
\]

\[
+ \beta[u_{m1}dC_{3,2} + (1 + \delta)^{-1}u_{C_{4,4}}dC_{4,2} + \alpha u_{C_{3,3}}dC_{3,1}] 
\]

\[
+ \lambda_2[u_{m1}dC_{3,2} + (1 + \delta)^{-1}u_{C_{4,4}}dC_{4,2} + \alpha u_{C_{3,3}}dC_{3,1}] 
\]

\[
+ \lambda_3[u_{m1}dC_{3,2} + (1 + \delta)^{-1}u_{C_{4,4}}dC_{4,2} + \alpha u_{C_{3,3}}dC_{3,1}] 
\]

\[
+ \lambda_4[...]
\]

\[
+...
\]

\[
d\psi = \lambda_1u_{m1}dC_{2,1} + [\lambda_1(1 + \delta)^{-1} + \lambda_1 \alpha \beta + \alpha \lambda_2]u_{C_{3,3}}dC_{3,1}
\]

\[
+ [\lambda_1 \beta + \lambda_2]u_{m1}dC_{3,2}
\]

(127)

In the private system we know that

\[
u_{m1} = \beta u_{m1}f_{T_{3,3}} = [(1 + \delta)^{-1} + \alpha \beta]u_{C_{3,3}}
\]

then

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\[ d\psi = \lambda_1 \beta u_{C^{m^*}_{t,2}} C_{t,2}^m dC_{2,1}^m \]
\[ + [\lambda_1 \beta u_{C^{m^*}_{t,2}} f_{T_2} T_2 + \frac{\alpha \lambda_2 \beta u_{C^{m^*}_{t,2}} f_{T_2}^m}{(1 + \delta)^{-1} + \alpha \beta}] dC_{3,1}^m \]
\[ + [\lambda_1 \beta + \lambda_2] u_{C^{m^*}_{t,2}} dC_{3,2}^m \]
\[ = \lambda_1 \beta u_{C^{m^*}_{t,2}} C_{t,2}^m dC_{2,1}^m \]
\[ + \beta u_{C^{m^*}_{t,2}} f_{T_2} T_2 [\lambda_1 + \frac{\alpha \lambda_2}{(1 + \delta)^{-1} + \alpha \beta}] dC_{3,1}^m \]
\[ + [\lambda_1 \beta + \lambda_2] u_{C^{m^*}_{t,2}} dC_{3,2}^m \]
\[ = \lambda_1 \beta u_{C^{m^*}_{t,2}} [dC_{2,1}^m + f_{T_2} dC_{3,1}^m + dC_{3,2}^m] \]
\[ + \lambda_2 u_{C^{m^*}_{t,2}} [\frac{\alpha \beta f_{T_2}}{(1 + \delta)^{-1} + \alpha \beta} dC_{3,1}^m + dC_{3,2}^m] \]

where substituting out \( f_{T_2} = \frac{[(1 + \delta)^{-1} + \alpha \beta]}{\beta \alpha} \), then

\[ dC_{3,2}^m + dC_{3,1}^m = f' (T_2) dT_2 + dS_2 = (\frac{1}{\beta \alpha (1 + \delta)}) + 1) dT_2 + dS_2 \]
\[ = dT_2 + dS_2 + \frac{1}{\beta \alpha (1 + \delta)} dT_2 > 0; \]
\[ dC_{3,2}^m + dC_{3,1}^m + dC_{2,1}^m = \frac{1}{\beta \alpha (1 + \delta)} dT_2 > 0 \]

So we get

\[ d\psi = \lambda_1 \beta u_{C^{m^*}_{t,2}} [dC_{2,1}^m + (\frac{1}{\beta \alpha (1 + \delta)}) + 1) dC_{3,1}^m + dC_{3,2}^m] \]
\[ + \lambda_2 u_{C^{m^*}_{t,2}} [dC_{3,1}^m + dC_{3,2}^m] \]
\[ = \lambda_1 \beta u_{C^{m^*}_{t,2}} [dC_{2,1}^m + dC_{3,1}^m + dC_{3,2}^m] + \lambda_1 \beta^2 \alpha (1 + \delta) u_{C^{m^*}_{t,2}} dC_{3,1}^m \]
\[ + \lambda_2 u_{C^{m^*}_{t,2}} [dC_{3,1}^m + dC_{3,2}^m] \]

We know that \( u_{C^{m^*}_{t,2}} [dC_{2,1}^m + dC_{3,1}^m + dC_{3,2}^m] > 0; u_{C^{m^*}_{t,2}} [dC_{3,1}^m + dC_{3,2}^m] > 0 \), then so long as \( dC_{3,1}^m > 0 \), then \( d\psi > 0 \).

Therefore, suppose we remove some units of \( C_{t,1}^m \) to \( T_2 \) and \( S_2 \), other variables outside time \( t = 2, 3 \) and the total resources in each period are kept the same, then welfare can be improved so that social optimum cannot be achieved through the private first order conditions.

**Case (2)**

Suppose we remove some units of \( C_{t,1}^m \) to \( T_2 \) and \( S_2 \) \((d(C_{t,1}^m) < 0, d(S_2) > 0, d(T_2) > 0)\), other variables as well as the aggregate resources outside time \( t, t+1 \) are kept at NE level, then what are the impacts of the reallocations.
on social welfare function? Importantly, the chain effects are considered here, meaning that reallocation has impacts on the lifetime utilities of all previous generations.

Differentiating welfare function $\psi$ wrt the change in $C_{t-1}^m$, $C_{t+1, t-1}^0$, $C_{t+1, t}^m$, subject to the changes in the aggregate resources at $t$ and $t+1$

$$d(C_{t-1}^m) + d(S_t) + d(T_t) = 0$$

where, $d(C_{t-1}^m) < 0, d(S_t) > 0, d(T_t) > 0$

$$dC_{t+1, t}^m + dC_{t+1, t-1}^0 = f'(T_{t, t})dT_{t, t} + dS_t$$

where $d(T_t)$ has an intertemporal impact on $f'(T_{t, t})dT_{t, t}$; $d(S_t)$ has the same intertemporal impact on the aggregate resource at $t+1$.

The effects of reallocation on $\psi$ are

$$d\psi(U_1, U_2, \ldots U_{t-1}) = \lambda_1[dU_1] + \sum_{t=3}^{\infty} \lambda_{t-1}[dU_{t-1}]$$

$$= \lambda_1[u_{C_{t-1}^m} dC_{t-1}^m + (1 + \delta)^{-1} u_{C_{t-1}^m} dC_{t-1}^o + \beta [u_{C_{t-1}^{C_2}} dC_{t-1}^{C_2} + (1 + \delta)^{-1} u_{C_{t-1}^{C_2}} dC_{t-1}^{C_2} + \alpha u_{C_{t-1}^{C_2}} dC_{t-1}^{C_2} + \beta \ldots$$

$$+ \beta [u_{C_{t-1}^{C_4}} dC_{t-1}^{C_4} + (1 + \delta)^{-1} u_{C_{t-1}^{C_4}} dC_{t-1}^{C_4} + \alpha u_{C_{t-1}^{C_4}} dC_{t-1}^{C_4} + \beta \ldots$$

$$+ \beta [u_{C_{t-1}^{C_3}} dC_{t-1}^{C_3} + (1 + \delta)^{-1} u_{C_{t-1}^{C_3}} dC_{t-1}^{C_3} + \alpha u_{C_{t-1}^{C_3}} dC_{t-1}^{C_3} + \beta \ldots$$

$$\ldots$$

$$+ \lambda_{t-1} [u_{C_{t-1}^m} dC_{t-1}^m + (1 + \delta)^{-1} u_{C_{t-1}^m} dC_{t-1}^o + \alpha u_{C_{t-1}^m} dC_{t-1}^o + \beta \ldots$$

$$+ \beta [u_{C_{t-1}^{C_2}} dC_{t-1}^{C_2} + (1 + \delta)^{-1} u_{C_{t-1}^{C_2}} dC_{t-1}^{C_2} + \alpha u_{C_{t-1}^{C_2}} dC_{t-1}^{C_2} + \beta \ldots$$

$$+ \lambda_t [u_{C_{t+1, t}^m} dC_{t+1, t}^m + (1 + \delta)^{-1} u_{C_{t+1, t}^m} dC_{t+1, t}^o + \alpha u_{C_{t+1, t}^m} dC_{t+1, t}^o + \beta \ldots]$$
\[ d\psi = \lambda_1 u_{C_{2,1}}^{m_t} \frac{dC_{2,1}}{t} \]
\[ + [\lambda_1 (1 + \delta)^{-1} + \lambda_1 \beta + \lambda_2 \alpha] w_{C_{2,1}}^{o_t} \frac{dC_{2,1}}{t} \]
\[ + [\lambda_1 \beta + \lambda_2] u_{C_{3,2}}^{m_t} \frac{dC_{3,2}}{t} \]
\[ + [\lambda_1 \beta (1 + \delta)^{-1} + \lambda_2 (1 + \delta)^{-1} + \lambda_2 \alpha + \lambda_3 \alpha] w_{C_{4,2}}^{o_t} \frac{dC_{4,2}}{t} \]
\[ + [\lambda_1 \beta^2 + \lambda_2 \beta + \lambda_3] u_{C_{5,3}}^{m_t} \frac{dC_{5,3}}{t} \]
\[ + [\lambda_1 \beta^2 (1 + \delta)^{-1} + \lambda_2 (1 + \delta)^{-1} + \lambda_3 (1 + \delta)^{-1} + \lambda_1 \beta^3 + \alpha_2 \beta^2 + \alpha_3 \beta + \lambda_4 \alpha] \frac{dC_{5,3}}{t} \]
\[ + [\lambda_1 \beta^3 + \lambda_2 \beta^2 + \lambda_3 \beta + \lambda_4] u_{C_{6,4}}^{m_t} \frac{dC_{6,4}}{t} \]
\[ + [\lambda_1 \beta^t + \lambda_2 \beta^{t-3} \ldots + \lambda_3 \beta^{t-3} + \lambda_4 \beta^2 + \lambda_5 - \lambda - 1] u_{C_{7,4}}^{m_t} \frac{dC_{7,4}}{t} \]
\[ + [\lambda_1 \beta^{t-3} (1 + \delta)^{-1} + \lambda_2 \beta^{t-4} (1 + \delta)^{-1} + \lambda_3 \beta^{t-4} (1 + \delta)^{-1} + \lambda_4 \beta^{t-4} (1 + \delta)^{-1} + \lambda_5 \beta^{t-5} \lambda_5 \beta^{t-5} \ldots] u_{C_{8,5}}^{m_t} \frac{dC_{8,5}}{t} \]

where \( dC_{2,1}^{m_t} = 0, dC_{3,1}^{m_t} = 0 \ldots dC_{t-2}^{m_t} = 0, dC_{t+2}^{m_t} = 0 \ldots \) (other variables outside time \( t, t + 1 \) are fixed and set at NE level).

Therefore,

\[ d\psi = [\lambda_1 \beta^t + \lambda_2 \beta^{t-3} \ldots + \lambda_3 \beta^{t-3} + \lambda_4 \beta^2 + \lambda_5 - \lambda - 1] u_{C_{7,4}}^{m_t} \frac{dC_{7,4}}{t} \]
\[ + (1 + \delta)^{-1} [\lambda_1 \beta^t + \lambda_2 \beta^{t-3} \ldots + \lambda_3 \beta^{t-3} + \lambda_4 \beta^2 + \lambda_5 - \lambda - 1] u_{C_{7,4}}^{m_t} \frac{dC_{7,4}}{t} \]
\[ + \alpha [\lambda_1 \beta^{t+1} - \lambda_2 \beta^{t-3} + \lambda_3 \beta^{t-3} + \lambda_4 \beta^2 + \lambda_5 - \lambda - 1] u_{C_{7,4}}^{m_t} \frac{dC_{7,4}}{t} \]
\[ + [\lambda_1 \beta^{t+1} - \lambda_2 \beta^{t+1} + \lambda_3 \beta^{t+1} + \lambda_4 \beta^{t+1} + \lambda_5 - \lambda - 1] u_{C_{7,4}}^{m_t} \frac{dC_{7,4}}{t} \]
\[ = [\lambda_1 \beta^t + \lambda_2 \beta^{t-3} \ldots + \lambda_3 \beta^{t-3} + \lambda_4 \beta^2 + \lambda_5 - \lambda - 1] u_{C_{7,4}}^{m_t} \frac{dC_{7,4}}{t} \]
\[ + (1 + \delta)^{-1} u_{C_{7,4}}^{m_t} \frac{dC_{7,4}}{t} \]
\[ + [\lambda_1 \beta^{t+1} - \lambda_2 \beta^{t+1} + \lambda_3 \beta^{t+1} + \lambda_4 \beta^{t+1} + \lambda_5 - \lambda - 1] u_{C_{7,4}}^{m_t} \frac{dC_{7,4}}{t} \]
Given the private NE conditions,

\[
\begin{align*}
\frac{du_{C_{t-1}}'}{dt} &= \alpha u_{C_{t-2}}' + \beta f_{T_{t-1}}' \quad \text{for } t \geq 1, \quad u_{C_{t-1}}' = [(1 + \delta)^{-1} + \alpha \beta]u_{C_{t-1}}', \\
\frac{du_{C_{t}}'}{dt} &= \alpha u_{C_{t-1}}' + \beta f_{T_{t+1}}' \quad \text{for } t \geq 1, \quad u_{C_{t}}' = [(1 + \delta)^{-1} + \alpha \beta]u_{C_{t+1}}'.
\end{align*}
\]

\(d\psi\) becomes,

\[
d\psi = (1 + \delta)^{-1}[\lambda_1 \beta^{t-2} + \lambda_2 \beta^{t-3} + \ldots + \lambda_{t-2} \beta^2 + \lambda_{t-1}][\frac{dC_{m, t-1}}{dt} + (1 + \delta)^{-1}u_{C_{t+1, t-1}}' + \alpha \beta]dC_{m, t-1} + \alpha \beta u_{C_{t+1, t-1}}' \quad \text{for } t \geq 1,
\]

Given that \(d(C_{t-1}^m) < 0, d(S_t) > 0, d(T_t) > 0; d(C_{t-1}^m) + d(S_t) + d(T_t) = 0; d(C_{t+1, t}^m) + d(C_{t, t-1}^m) = f'(T_{t, t})dT_{t, t} + dS_t,

we get

\[
\begin{align*}
\text{(1)} & \quad dC_{t-1, t}^m + dC_{t+1, t-1}^m = f'(T_{t, t})dT_{t, t} + dS_t > 0 \\
\text{where, } f'(T_{t, t}) & > 1
\end{align*}
\]

\[
\begin{align*}
\text{(2)} & \quad dC_{t-1, t}^m + dC_{t+1, t-1}^m + dC_{t, t}^m = (f'(T_{t, t}) - 1)DT_{t, t} > 0 \\
\text{where, } f'(T_{t, t}) & > 1
\end{align*}
\]

So long as \((f'(T_{t, t}) - 1)DT_{t, t} - dC_{t+1, t}^m > 0 \text{ (DT}_{t, t} > (1 + \sigma)\alpha \beta dC_{t+1, t}^m), \text{ then equations (139) and (140) yield,}

\[
\begin{align*}
\text{(3)} & \quad dC_{t-1, t}^m + dC_{t+1, t-1}^m > 0
\end{align*}
\]

then,

\[
d\psi = (1 + \delta)^{-1}[\lambda_1 \beta^{(t-1)} - 2 + \lambda_2 \beta^{(t-1)} - 3 + \ldots + \lambda_{t-2} \beta^2 + \lambda_{t-1}][\frac{dC_{m, t-1}}{dt} + dC_{t+1, t-1}^m \quad \text{for } t \geq 1,
\]

\[
\begin{align*}
\text{+} \quad \lambda_{t-2} \beta^2 \quad \text{for } t \geq 1, \\
\text{+} \quad \lambda_{t-1}^2 \beta^0 \quad \text{for } t \geq 1
\end{align*}
\]

59 Let's take one numerical example, supposed \(dC_{t-1, t}^m = -1; dT_{t, t} = 0.5, dS_t = 0.5, f' = 1.5 \text{ then } dC_{t+1, t-1}^m + dC_{t+1, t-1} = f'(T_{t, t})dT_{t, t} + dS_t = 1.25. \text{ Therefore, it is possible to set } dC_{t+1, t}^m = 0.1, \text{ then } f'(T_{t, t}) - 1)DT_{t, t} - dC_{t+1, t}^m = 0.15 > 0.

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Proposition Suppose that $\lambda_{t-1} \geq 0$, $t \geq 2$, the private NE conditions for consumptions, trainings and savings cannot be a welfare optimum.

Taxation/Subsidy Having identified that the outcomes in the presence of pure altruism (externality) cannot be a social optimum, we now consider another possible solution. We consider government-implemented quotas or taxes as one instrument to restore optimal by imposing a tax/subsidy on the externality generating agent. This well known solution is Pigouvian taxation. The principle for the case of negative externality in the warm glow type of altruism is that we take the form of a per-unit subsidy for each unit of the externality a person lose (or, the form of a per-unit tax for each unit of the externality a person gain). To this effect, we look at the donor decisions in the warm glow type of altruism by having subsidies. Suppose that government gives the middle aged donor a subsidy of $t_R$ per unit of free gift to the old and a subsidy of $t_T$ per unit of training investment to the young. Then it is not difficult to see a subsidy of $t_{R,t-1} = \frac{\lambda_{t-2}}{\lambda_{t-1}}(1 + \delta)^{-1}u^o_{C_{t,t-1}}$ and subsidy of $t_{T,t-1} = \frac{\lambda_t}{\lambda_{t-1}}u^o_{C_{t+1,t-1}}f_{T,t}$ will implement the optimal level of the externality. Like all Pigouvian taxes note these are endogenous, the right hand of the expressions for the tax rates depend on the endogenous values of consumption. Indeed, the donor will maximise his/her utility in response to those two subsidies to choose the level of $C_{t,t-1}^m$ and the transfers as,

$$
\text{obj}_{t-1} = \max_{ R_{t,t-1} \geq 0, S_{t,t-1} \geq 0, T_{t-1} \geq 0 } \{ u^o(C_{t,t-1}^m) + (1 + \delta)^{-1}u^o(C_{t+1,t-1}^o) \} + [\alpha v(C_{t,t-2}^o) + \beta v(f(T_{t-1}))]
$$

s.t. \{ f(T_{t-1,t-1}) + t_{R,t-1} + t_{T,t-1} - T_{t-1} - T_{t,t-1} - S_{t,t-1} \geq C_{t,t-1}^m \\
R_{t+1,t} + S_{t+1,t-1} \geq C_{t+1,t-1}^o \\
R_{t+1,t} + S_{t+1,t-1} \geq C_{t+1,t-1}^o \\
R_{t,t-1} + S_{t-1,t-2} \geq C_{t,t-2}^o \\
\}

After substituting out the constraints, the objective function becomes

$$
\text{obj}_{t-1} = \max_{ R_{t,t-1} \geq 0, S_{t,t-1} \geq 0, T_{t-1} \geq 0 } \{ u^o(f(T_{t-1,t-1}) + t_{R,t-1}R_{t,t-1} + t_{T,t-1}T_{t,t-1} - R_{t,t-1} - T_{t,t-1} - S_{t,t-1}) + (1 + \delta)^{-1}u^o(R_{t+1,t} + S_{t,t-1}) \}
$$

$$
+ [\alpha v(R_{t,t-1} + S_{t-1,t-2}) + \beta v(f(T_{t,t}))] + t_{R}R_{t,t-1} + t_{T}T_{t,t}
$$

For the interior solutions, we have

$$
Foc, R_{t,t-1} : u'^{C_{t-1}^m}_{t,t-1} = u'^{R_{t,t-1}}_{t,t-1} + \alpha v'^{C_{t-2}^o}_{t,t-2}
$$

$$
Foc, T_{t,t} : u'^{C_{t-1}^m}_{t,t-1} = u'^{T_{t,t}}_{t,t-1} + \beta v'^{f_{T_{t,t}}}_{t,t}
$$

Comparing to the focs in the social system,

$$
Foc, R_{t,t-1} : u'^{C_{t}^m}_{t-1} = \frac{\lambda_{t-2}}{\lambda_{t-1}}(1 + \delta)^{-1}u'^{C_{t-2}}_{t,t-2} + \alpha v'^{C_{t-2}}_{t,t-2}
$$

$$
Foc, T_{t,t} : u'^{C_{t}^m}_{t-1} = \beta v'^{f_{T_{t,t}}}_{t,t} + \frac{\lambda_{t}}{\lambda_{t-1}}u'^{C_{t}^m}_{t+1,t}f'^{T_{t,t}}_{t,t}
$$

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Therefore, if we set \( u_{C_{t+1}}' t, t-1 = \frac{\lambda_{t-1}}{\lambda_t}(1 + \delta)^{-1} u_{C_{t+1}}' t, t-2 \) and \( u_{C_{t+1}}' t, t-1 = \frac{\lambda_{t-1}}{\lambda_t} u_{C_{t+1}}' t, t, f_{t, t} \), then \( t_{R, t-1} = \frac{\lambda_{t-2}}{\lambda_t}(1 + \delta)^{-1} \frac{u_{C_{t+1}}' t, t-2}{u_{C_{t+1}}' t, t-1} \) and \( t_{T, t-1} = \frac{\lambda_{t-1}}{\lambda_t} \frac{u_{C_{t+1}}' t, t, f_{t, t}}{u_{C_{t+1}}' t, t-1} \). So, note that a subsidy approach is equally effective in achieving social optimum. However, the social planner must have a great deal of information about the benefits and costs of the externalities for the generations to set the optimal level of subsidy.

### 4.4 Conclusion

In this chapter, we developed analytic frameworks within the household decision theory to understand intergenerational transfers and education investment in an altruistic-OLG setting with three generations. For the nature of the household model, we consider a non-cooperative household environment, in which three generations (the O, the M and the Y) live in the same household and only the middle aged generation is an independent decision maker and altruistic. However, each individual lives three periods so the roles of being a decision maker and altruism change over time. The intergenerational (or intra-household) transfers are determined by the interdependent preferences between generations. The intergenerational transfers are normally in the forms of gifts and training investment and the amount of the transfers vary with the degree of altruism. For the incentives to do transfers, we start from a mixture between the censored altruism (warm glow/social expectation) and semi-altruism (bounded rationality), and then move to the pure altruism (pure love) to study whether the private Nash equilibrium in intertemporal transfers can be a social optimum. For the behaviour of the system over time, we apply OLG in a private system and in a time-discrete environment, in which three generations live in the same household with a new born young generation at each date and there are economic interactions between generations because of altruism.

For the basic setting of the economic environment, we follow Samuelson’s (1958) consumption loan story but study the intergenerational transfers in a new box: family altruism. For the purpose of understanding the role of intra-household transfers, we assume that the uneducated Y and retired O do not work and the M is the only working generation. The income of the M is the main financial resource for the whole family, the O relies on the gift sent by the M and the amount of good stored (or the amount of money left in the pot) in the previous period. We notice that it is possible to build a self-funded household without any financial instruments. The economic system won’t break down as soon as there are altruism and intergenerational transfers.

As we indicated earlier, each middle generation makes choices independently and each has perfect foresight on other’s choices. In this chapter, the private transfers between three generations within the household is actually a redistribution process. This redistribution process is undertaken through a private institution, that is the household, rather than by public allocation activity, such as a fiscal process. This raises questions for welfare optimum. Without...
the public allocation activity, can a social optimum be achieved through altruistic transfers? Can redistribution within the household be sufficient for the attainment of social optima? In terms of the question for welfare optimum in an OLG environment, we can study it in a sub-period intertemporal format, in the sense that a central planner maximizes some adjacent agents’ lifetime utilities over aggregate resources in each period, and other state variables before the beginning of sub-period and after the end of sub-period are set at their full NE values. Within the subperiod, the central planner redistributes consumptions and chooses training.

For the censored altruism case, we find that the social optimum (for a reasonable but particular social welfare function form) cannot be achieved through the private NE. There are externalities which are not accounted for in the private system. In order to correct this, we move to pure altruism and turn for help to taxation/subsidy. Interestingly, we find that the private NE conditions in the pure altruism are not enough to help the private system to obtain a social optimum. We have to use subsidy or taxation as one instrument. Besides, we also propose a preliminary idea below: a family loan, to force everyone to commit and no one to deviate.

In general, this chapter develops Samuelson’s social security idea but under altruism. In particular, this chapter explains the possibility of building a self-funded household with intergenerational transfers and education investment. In an economic environment without financial instruments, the existence of altruism between family members is one of the sufficient conditions to make transfers happen. Under certain degree of government intervention (e.g. tax rate or subsidy), the welfare optimum could be achieved by the private system.

4.5 Some further thoughts

This chapter only explores the link between the intergenerationally altruistic transfers and education investment. Some other aspects could be investigated in the future. There could be different types of children: well-behaved children and naughty children. Parents may have asymmetric information on children’ type. The well-behaved children would give true information to their parents and give repayment when their parents get old. The naughty children would report false information to their parents and may not give repayment when their parents get old. We could have a family loan with rotten kids story: within this three period environment, parents could lend money to their kids in the first period and the kids pay back the loan in the second period when their parents become old. In this story, we could bring pooling strategy, or IC (incentive compatibility constraint) or punishment into the modelling part to understand what is the best response if the young generation is not committed or if the parent does not have complete information on the children. (i) One possible approach that the parent can use is a pooling strategy, making all child types do the same amount of repayment. But this strategy leads to a low education investment because parents would like to get a secured repayment which is a low amount of repayment made by the
naughty children. Knowing this, parents would make a low education investment. The well-behaved children would get a low amount of education investment as well. This pooling strategy is not efficient as the well-behaved children cannot get a high education investment, may leading to some welfare loss. (ii) We can also set an appropriate incentive compatibility constraint to prevent naughty children from cheating (e.g. Browning, 2009). (iii) We can set up a punishment, such as a big financial fine or social reputation. A heavy financial fine will be levied so long as the children cheat. All these are just preliminary thoughts, which need to be carefully designed when we do the modelling.

Empirically, there are also a few possible research topics. For instance, (i) we could test for altruistic behavior using a lab experiment (e.g. Andreoni et al, 2007); (ii) we could test the importance of love (e.g. time spent on child care) and money (e.g. education investment) on the outcome of children’s HC and investigate whether these two could be substitute or complementary. The outcome of the HC might be different between the child who was sent to the private school but not receiving enough child care and the child who was sent to public school but receiving enough child care.
5 The Patterns of Intergenerational Educational and Occupational Mobility for Rural-urban Migrants in China

5.1 Introduction

This chapter tries to answer question 5. In order to do this, we study intergenerational mobility in education and occupation for rural-urban migrants in a developing country, China. Over the last few decades, China, as the world’s most populous nation, and the unprecedented rise in its living standards and economic growth together with the transformation in the Chinese economy is eye-catching. However, immobility limits equal opportunities for the next generation. Especially, the rural-urban migrants, as one minority group in the urban society, witness the dramatic changes both in rural and urban areas. Their parents are normally rural farmers and their children are sometimes educated and employed in urban areas. This implies that migrants could change their social categories through migrating from rural to urban areas and their children can also change social categories by being taken by their parents and educated in urban areas. But most of migrants’ parents did have the opportunities to use migration as a channel to change their social categories. This makes Chinese rural-urban migrants an very interesting group to understand the changes in mobility between rural-urban migrants, their parents and their children in China. It is most likely that the migrants’ children are in a higher social category than the migrants and migrants’ parents. Because they benefit not only from migration but also from better education in the urban area. In addition, based on the positive correlation between intergenerational mobility and equality ("Great Gatsby Curve", Wikipedia, 2010). We suspect the inequality could be reduced through rising mobilities.

With complete mobility each generation can have equal economic opportunity. In Rawls’s (1971) social justice world, equal economic opportunities can be interpreted that each individual when can be born with different resources but should be entitled to equal opportunities. Sen (1983) also claims that individuals come with different capabilities but every one should be given equal opportunities. Under the equal opportunities condition, the existence of inequality is acceptable (Sen, 1983). The economic opportunities can be mirrored by the pattern of intergenerational mobility. If each individual has the same chance of being placed in any economic class irrespective of the position of their parents then all individuals have the same life chances.

One most commonly known concept to study intergenerational mobility is using permanent income. This method requires information on individuals’ lifecycle income (e.g. income levels for 60 years). Obviously, this requires a large panel data set. Most researchers use the average value of lifecycle income during certain time intervals (e.g. aged 40-50) as a proxy for permanent income. This methodology can generate biased information. For instance, one successful entrepreneur can decide to retire before he turns 40. Then the permanent income for this entrepreneur
cannot be captured by the average value of income during ages 40 and 50. In order to eliminate the biases and tackle
the data limitation problem, we use individuals’ education level and occupation as two proxies for permanent income
to capture social category changes for each generation. Assuming most migrants do not update their educational
levels and occupation during their lifetime (this is a reasonable assumption for rural-urban migrants in China),
then we believe that having a better education signals a better human capital level for an individual, which further
indicates a good occupation and a high level of permanent income (Fig 43).

Consequently, research questions resulting from this context are: How do the patterns of IGM for rural-urban
migrants change during China’s dynamic evolution? In particular, will the richest transmit the same social status
to the next generation? By contrast, will the poorest be unable to help their children to gain social promotion? As
well as this, will the patterns of IGM persist over time across generations? Is there any time trend in the pattern
among different generations? Will the distributions of education and occupation converge to stationary states over
time? If so, how long will they take to converge?

To answer the questions above especially the changes in the patterns of IGM, we have to identify the relative
position changes in education and occupation across generations within the same household. In order to do that,
we study the patterns through absolute and conditional mobility across three generations: grandparent, parent and
child on 2008 micro-data. Absolute mobility measures the observed social category change between parent and
child whatever its causes. This mobility is a result of parents’ characteristics and of economic environment, macro
economic development, political shocks, characteristics of each generation etc. To describe this, we use transition
matrices. Through this methodology, we look at the chance of a child moving from parents’ social category to
a different one. Conditional mobility takes account of multiple causes of mobility, and applying this empirically
with a particular model specification, allows identification of the effects of different causes. In particular, it allows
isolation of the effects of parental education/occupation on education/occupation of the child by controlling for other
influences. The mobility is more due to micro characteristics of parents or some individual personalities. Especially,
we consider a causal link between parental education level (occupation) and child’s education level (occupation).
The ordered probit econometric model helps us study the causes of the relative position change between generations. The results from transition matrices show that there are relatively high chances of moving from the bottom to the median level but apart from this, movements are restricted to a small shift up or down. The results from the ordered probits show that there is educational/occupational immobility with significant effects of education/occupation of the older generation.

Having established the transition matrices, the dynamics of the education and occupation distribution can be analysed. Treating the process as a Markov Chain, we find that typically for both variables after ten generations or more the distribution has converged to its stationary state. The interpretation is that if there is no policy initiative to shift the intergenerational immobility revealed by the transition matrices then China will remain with distributions showing inequality and opportunities for the young depending on their parental background.

Having said that, the contributions of this chapter are fourfold: (i) we use IGM in education and occupation as two proxies to study the IGM in permanent income; (ii) we use transition matrices and ordered probits to capture the relative position change; (iii) in particular, we isolate the effects of parental education/occupation on education/occupation of the child to generate a predicted mobility. And we compare this predicted mobility with the absolute mobility in occupation. We find that if all individuals had identical characteristics and environmental variables (at the mean) the chance of moving upwards would slightly increase. This suggests that heterogeneity in the environment might be one cause of the lack of equality of opportunity; (iv) we use Markov Chains to calibrate time periods of distributions converging to a stationary state to make some possible policy suggestions. If there is no policy initiative to shift the intergenerational immobility, the society stays unequal (in the stationary distribution) and the young generation won’t have equal opportunities to change their social categories.

This chapter is structured as follows. Section 2 provides a brief literature review. Section 3 presents the basic theoretical framework and measurement. Section 4 describes the data and summarizes the statistics. Section 5 introduces the methodology Section 6 interprets the results and predict the dynamics of IGM through the Markov chain. The final section concludes.

5.2 A brief literature review

Labar (2011) uses an ongoing longitudinal survey from the 1991, 1993, 1997, 2000 and 2004 rounds of the China Health and Nutrition Survey (CHNS) covering nine provinces to estimate the intergenerational educational and income mobility in China. The IGM is studied by the transition matrix for absolute and conditional mobility. The main findings in her paper are that there is upward mobility for lower education levels and at the upper ends there is some mobility in both directions. For income, there is some mobility at middle income levels but the rich and
the poor tend to be immobile between generations. Income mobility at middle income levels could be due to the Hukou reforms in China allowing a broader access to higher paid jobs. For conditional mobility, she finds that "the parents’ educational attainment and income are playing a significant role in the determination of the child’s level of education". Over time (1991-2004), the impact of parental education on that of the child is falling.

Meng et al (2010) use Urban Household Education and Employment Survey 2004 (UHEES) and the Urban Household Income and Expenditure Survey 1986-2004 (UHIES) to estimate intergenerational income elasticity for urban China and find that elasticities are 0.74 for father-son, 0.84 for father daughter, 0.33 for mother son and 0.47 for mother-daughter. For the elasticities, they claim that educational level of the parents, parents’ social network, the CPC (the Communist Party of China) status of the parents are important. The coefficient between parents’ and children’s education ranges from 0.44 to 0.49 for father-son, father-daughter, mother-son and mother-daughter. Parent’s social networks are important for children’s education in the sense that parents with better social networks can provide their children with access to better opportunities in the education and labor markets. The CPC membership plays a similar role as social network. In this paper, due the unavailability of panel data, we will not estimate an elasticity for intergenerational mobility. But we will look at the absolute and conditional mobility also for father-son, father-daughter, mother-son and mother-daughter.

Quheng, Gustafsson & Li (2012) use individual and household level data in China for 1995 and 2002 (supplemented by additional individual income data for 1991-2 and 200-2001) to estimate the IGM of income for parent and child groups differentiated by gender (e.g. fathers and sons or mothers and daughters). They allow for a selection effect of children who are at home on IGM and find this significant and negative. The decision to live at home or away allows for age, ethnic and regional effects. They find higher IGM elasticities than Labar and also higher than the studies reported below. Although the gender differences are small, there is a slight tendency for parent-son elasticities to be higher.

Blanden et al (2013) use two prominent longitudinal panel survey data sources, the 1970s Birthday Cohort Study (BCS) for UK and the 1960s Panel Study of Income Dynamics (PSID) for the US to examine the intergenerational mobility for both countries. In their study, children’s income is a function of educational level, occupation, parents’ income, labor market attachment, marital status and health. Except occupation and the peer effect of social network, our estimation equation for intergenerational income mobility is consistent with the estimated equation in their paper. As a result of higher returns to education and skills in the US, the educational level of the children is relatively more important than it is in UK. By contrast, children’s occupation is more important in UK than in the US. Children’s health, marital status and labor market attachment have little explanatory power in explaining the
intergenerational income mobility in the US and UK. Overall, the results support a strong link between parent’s income and children’s income in both countries.

Lefgren, Lindquist & Sims (2012) identify the different impacts on the IG income elasticity between children and parents due to parental human capital and parental financial resources, finding that most of the effect of parents is due to their human capital and not their financial resources.

Zyllberberg (2007) points out that intergenerational transmission is an ongoing process between successive generations so that the Markovian mobility transition matrix implies that generations prior to the parents will have an impact on the relative position of children. He applies this to PSID data for the US on the occupational group of successive generations, and finds much more persistence of occupation of successive generations than would be modelled by a one period link between successive generations.

5.3 Basic framework

5.3.1 Lifecycle effect

Permanent income could be captured by life cycle occupation and education as those three are closely related. We first look at the link between life cycle occupation and permanent income. Different occupations have very different patterns of income with age and also different permanent incomes. A real problem in measuring IGM in income is the life cycle effect, because we do not have data on child and parent incomes at each age in their life cycle (which would require a 60 year plus panel). So researchers resort to using age as a regressor in child against parent income regressions, or some short term average of child and/or parent income to proxy permanent income. However different occupations are associated with different permanent incomes: the permanent income of a banker is higher than that of a labourer although the life cycle pattern of incomes may differ between the two. If IGM in income is really about permanent income or income comparisons at the same stage of the life cycle between parent and child then looking at the transmission of occupation to child could be a good approach which is feasible with the available data. The life cycle effects of occupation on income, and also the effects of occupation on permanent life cycle income are seen in Fig 44-47. Apart from permanent incomes for "personal care assistants, nursing assistants for families" and "owner of private sector", nearly all the life cycle incomes have hump shapes, but each reaches the peak at a different age period. "Managers" have fairly high life cycle income but may forgo income at a young age to pay for the entry cost. While "cleaners and laundry workers" receive high income at a

60 The U shape of permanent income for individuals doing "personal care assistants, nursing assistants for families" can be explained by special features in "personal care" sector. Those aged 16-25 are highly demanded as babysitters or tutors by families in China. Those aged 46-55 are signaled as good chefs and elderly carers by Chinese families based on well recognized characteristics of the age profiles in China. Those aged between 26 and 45 are considered as busy working middle generation who stay on the demand side of this "personal care" sector.
young age and "protective service workers" face falling income after age 26-35. Individuals who work in the service or manufacturing industries have fairly stable income. According to the mean value of lifecycle income (Table 8), we group those 24 occupational categories into 4 groups. Those in more "working class occupations" (Fig 44 and ??) show age related patterns that fall with age from an early peak on and lower means than those in the "more middle/higher class " occupations (Fig 46 and Fig 47), who have a higher mean over the life cycle. In the high mean group (Fig 47), the "owner of private sector" has a falling income over life time although he/she has the highest level of permanent income. This phenomenon is consistent with our argument against the methodology used by most researchers, who take the average value of income during certain lifetime periods as a proxy for permanent income. In this case, the average value of income during 40 to 50 years for the "owner of private sector" is much lower than his/her real lifetime income. Instead, if we use occupation as a proxy for permanent income can generate better information.

In addition, we believe there is a link between occupational choice and education of an individual. Apart from "self-employed" and "owner of private sector", those in higher mean income occupations tend to have higher education (Fig 48). The "self-employed" individuals and "owner of private sector" have high levels of permanent income but low educational levels because of longer working hours per week (Fig 49). Rural-urban migrants obtain high levels of permanent income through either better education, or good occupation or longer hours of work.

And for Chinese rural-urban migrants, the link between education level and permanent income is important for IGM but not as strong as the link between occupation and permanent income, which is due to the features

Table 8: Occupation grouping and income

<table>
<thead>
<tr>
<th>Occupation</th>
<th>16-25</th>
<th>26-35</th>
<th>36-45</th>
<th>46-55</th>
<th>Average permanent income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food trades assistants, kitchenhands</td>
<td>1015</td>
<td>1195</td>
<td>730</td>
<td>783</td>
<td>980.75</td>
</tr>
<tr>
<td>Cleaners and laundry workers</td>
<td>1146</td>
<td>1064</td>
<td>1042</td>
<td>890</td>
<td>1035.5</td>
</tr>
<tr>
<td>Hospitality workers in restaurant</td>
<td>1112</td>
<td>1184</td>
<td>1105</td>
<td>1170</td>
<td>1142.75</td>
</tr>
<tr>
<td>Waiter, waiter, and food service workers</td>
<td>1056</td>
<td>1457</td>
<td>1446</td>
<td>1031</td>
<td>1247.5</td>
</tr>
<tr>
<td>Recycling and rubbish collectors</td>
<td>933</td>
<td>1322</td>
<td>1475</td>
<td>1291</td>
<td>1255.25</td>
</tr>
<tr>
<td>Protective service workers for factory garage or for local residence (group1)</td>
<td>1286</td>
<td>1427</td>
<td>1279</td>
<td>1102</td>
<td>1273.55</td>
</tr>
<tr>
<td>Repair, appliance repairman</td>
<td>1217</td>
<td>1813</td>
<td>1204</td>
<td>916</td>
<td>1295.5</td>
</tr>
<tr>
<td>Personal carer assistants, nursing assistants for family</td>
<td>1045</td>
<td>1254</td>
<td>1087</td>
<td>1357</td>
<td>1385.75</td>
</tr>
<tr>
<td>Other service workers</td>
<td>1304</td>
<td>2042</td>
<td>1411</td>
<td>900</td>
<td>1414.25</td>
</tr>
<tr>
<td>Handlers and delvers</td>
<td>1467</td>
<td>1529</td>
<td>1383</td>
<td>1346</td>
<td>1431.25</td>
</tr>
<tr>
<td>Maintenance and production services workers</td>
<td>1497</td>
<td>1576</td>
<td>1307</td>
<td>1390</td>
<td>1442.5</td>
</tr>
<tr>
<td>Other factory process workers (group2)</td>
<td>1355</td>
<td>1583</td>
<td>1570</td>
<td>1416</td>
<td>1481</td>
</tr>
<tr>
<td>Chefs and butchers</td>
<td>1300</td>
<td>1939</td>
<td>1443</td>
<td>1254</td>
<td>1484</td>
</tr>
<tr>
<td>Personal service workers (SPA, hair dresser, tourists guide)</td>
<td>1432</td>
<td>2200</td>
<td>1626</td>
<td>800</td>
<td>1514.75</td>
</tr>
<tr>
<td>Sales workers</td>
<td>1310</td>
<td>1542</td>
<td>1642</td>
<td>1606</td>
<td>1523</td>
</tr>
<tr>
<td>Machinery operators and factory process workers</td>
<td>1442</td>
<td>1638</td>
<td>1615</td>
<td>1514</td>
<td>1552.25</td>
</tr>
<tr>
<td>Farmers</td>
<td>1750</td>
<td>1810</td>
<td>1800</td>
<td>1200</td>
<td>1640</td>
</tr>
<tr>
<td>Clerical and admin workers</td>
<td>1445</td>
<td>1758</td>
<td>1714</td>
<td>1487</td>
<td>1601</td>
</tr>
<tr>
<td>Construction labourers</td>
<td>1624</td>
<td>1749</td>
<td>1671</td>
<td>1591</td>
<td>1658.75</td>
</tr>
<tr>
<td>Automobile drivers, delivery drivers (group3)</td>
<td>1600</td>
<td>1972</td>
<td>1805</td>
<td>1775</td>
<td>1788</td>
</tr>
<tr>
<td>Technical Professionals</td>
<td>1373</td>
<td>2206</td>
<td>1893</td>
<td>1870</td>
<td>16666667</td>
</tr>
<tr>
<td>Self-employed</td>
<td>2123</td>
<td>2381</td>
<td>2053</td>
<td>1905</td>
<td>2115.5</td>
</tr>
<tr>
<td>Managers</td>
<td>1688</td>
<td>2166</td>
<td>2815</td>
<td>2175</td>
<td>2211</td>
</tr>
<tr>
<td>Owner of private sector (group4)</td>
<td>3140</td>
<td>2567</td>
<td>2282</td>
<td>2071</td>
<td>2515</td>
</tr>
</tbody>
</table>

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Figure 44: Occupation group 1 (Bottom mean of permanent income)

Figure 45: Occupation group 2 (Low mean of permanent income)
Figure 46: Occupation group 3 (Middle mean of permanent income)

Figure 47: Occupation group 4 (High mean of permanent income)

Figure 48: Average education level of different occupation
of Chinese labor and "variations in hours of work". Most Chinese rural-urban migrants work long hours and are relatively lowly educated. So the effect of education on permanent income is less obvious. Meanwhile, the low educated ones can always compensate for a low wage rate by working more hours. Comparing Fig 48 and 49, in the high mean group, the "owner of private sector" and the "self-employed" individuals obtain high permanent incomes not through high levels of education but through more working hours. In the middle mean group, the "chefs and butchers" and the "personal services workers (APS, hair dresser, tourist guider)" compensate low educational levels by more working hours per week to gain decent permanent incomes. In the low mean group, individuals (e.g. fruit, veg and rice sales men) with very low educational levels need to work even harder to keep a basic living standard. This also suggests that if we have data on permanent incomes, then we should really look at permanent wage rates.

However, this is unavailable and following the facts stated above, IGM of permanent income can be quite well approached by studying intergenerational transmission of occupation and education.

5.3.2 Theory

Having stated the close link between permanent income, lifecycle occupation and education, we present a theoretical framework for interpreting the evidence from the literature on IGM. We will first use the theoretical framework based on IGM in income to guide us to analyse IGM in occupation and education. We begin with the Becker-Tomes (1979) model in which the intergenerational income mobility equation is formalized and follow a recent work done by Blanden et al (2013) who compare IGM between the US and UK. IGM in income can be summarised below.

The income of a child is a function of some personal characteristics of the child, education level, occupation and

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61Besides that, this is also supported by parents concern over the occupation that their children enter. Some of the more professional occupations involve long training post education and a steep profile in life cycle earnings. For example training to become a government employee has a high entry cost but yields subsequent high rewards. Transmission of these occupations is likely to be conditional on the current financial resources of parents which is further conditional on educational levels of parents.
the income level of his/her parents,

\[ y_{i,t} = y(E_{i,t}, O_{i,t}, X_{i,t}, y_{j,t-1}, \varepsilon_t) \]

where \( y_{i,t} \) is income level of the child who is born at \( t \); \( y_{j,t-1} \) is his/her parents income who were born at \( t - 1 \); \( E_{i,t} \) is the educational level of the child; \( O_{i,t} \) is the occupational category of the child; \( \varepsilon_t \) is income shock including idiosyncratic and macro shocks; \( X_{i,t} \) is the individual characteristics of the child; the interval between \( t \) and \( t - 1 \) is about 30 years.

Theories of IGM in education, such as parental investment in children, suggest several channels through which family circumstances may influence their children’s educational attainment. The pioneer of this field is Becker (1989) who assumes that parents are altruistic and choose the level of education investment through a unitary household framework. Becker believes that altruistic parents influence children’s human capital levels by education investment, home environment such as love and time spent with child and parental education. Moav (2005) models the dynamic evolution of intergenerational human capital level and finds that the human capital level of the child is influenced by his/her parents. Aydemir et al (2008) test the intergenerational educational mobility empirically. The basic idea of intergenerational educational mobility can be expressed as

\[ E_{i,t} = E(E_{j,t-1}, Z_{i,t}, Z_{j,t-1}, \theta_t) \]

where \( E_{i,t} \) is the level of educational level of the child; \( E_{j,t-1} \) is the level of educational level of the parent and is also a proxy for parent’s income (high educational level parents tend to have higher incomes. Those parents are more likely to invest more on child’s education); \( Z_{i,t} \) is the individual characteristics of the child; \( Z_{j,t-1} \) is the individual characteristics of the parent such as age; \( \theta_t \) is an educational shock including idiosyncratic and macro shocks. For simplicity, we exclude deterministic macro variables such as a time trend in GDP or in efficiency of the education system, although conceptually these could be easily estimated by having an age variable to catch the time effect.

As far as we know, theories of IGM in occupation have not been well established. The only relevant works are Zyllberberg (2007) and Xie & Killewald (2013), which mainly use transition matrices to look at absolute mobility for occupation. Following the concepts of Zyllberberg (2007) and Xie & Killewald (2013) and the framework of intergenerational educational mobility, we apply a similar framework for intergenerational occupational mobility. The mobility equation is written as,

\[ O_{i,t} = O(O_{j,t-1}, W_{i,t}, W_{j,t-1}, v_t) \]

where \( O_{i,t} \) is the level of occupational level of the child; \( O_{j,t-1} \) is the level of occupational level of the parent and is
also a proxy for parent’s income (parents with good occupations tend to have high incomes); \(W_{i,t}\) is the individual characteristics of the child; \(W_{j,t-1}\) is the individual characteristics of the parent; \(v_t\) is an occupational shock.

Substituting out occupation and education equations in the income equation, we can see a close link\(^{62}\) between permanent income, lifecycle education and occupation. Therefore, the IGM in income can be studied through IGM in education and occupation.

5.3.3 Measuring IGM

In line with the theories above, in this section, we provide an attempt to measure IGM. Especially, we are interested in the distribution of the intergenerational occupational and educational levels to describe the extent of mobility for different generations and the chances that an individual can move from one part of the distribution to another to identify the mobility pattern between generations.

- Distribution

For different generations, the whole distribution for income and educational levels (Figure 50) as well as the extent of mobility for different generations can be different. The differences are caused by the change in individual characteristics, macro trends, the effect of parents and the shocks. The first, second or higher order moments can be changed due to those changes. For instance, (i) an increase in public education funding for one generation can shift the entire distribution to the right, then the inequality (equal opportunity) can be kept the same if the shape of the distribution stays the same but the mobility (relative position change) cannot be identified without having micro information on the rank of each individual between quantiles. (ii) One generation can be more diligent than the other one. Individuals put more effort on their studies and work. Then the mean values of income and educational level are higher than the previous ones. The variance can be reduced as well when everyone moves to the median or mean level due to effort. In this scenario, the mobility degree can be higher for this generation. (iii) A financial shock can systematically damage the whole economy. If everyone receives the same shock and the income level is decreased roughly at the same percentage, then the entire distribution will be shifted to the left but the variance of log income stays the same. This won’t affect mobility degree. In addition, an idiosyncratic shock at individual level can modify the variation and skewness of one generation, can lead to a change in mobility degree.

- Who moves where

However, the whole distribution shift won’t help us identify the relative position change between parent and child within the same household. In order to identify particular position change from one distribution to another

\[^{62}y_t = y(E_{t-1}, O_{t-1}, X^I_t, X^P_t, \varphi_t),\] where permanent income \((y_t)\) of generation \(t\) can be proxied by education \((E_{t-1})\) and occupation \((O_{t-1})\) of previous generation \(t-1\); \(\varphi_t\) is just a random shock at time \(t\). Here, we use \(E_{t-1}\) and \(O_{t-1}\) as two proxies for parents’ income.
Variables of interest, for absolute and conditional mobility, are education $E_{h,t}$ and occupation $O_{h,t}$ for person $h$ of generation $t$. Individuals are divided into groups say $I$ for the children and $J$ for their parents. $N_I$ is number of children in group $I$, $N_J$ is the number of parents in group $J$. $n_{IJ}$ is number of children in $I$ whose parents were in $J$.

1 Absolute mobility

For absolute mobility, the position change in social category for the children is caused not only by characteristics of their parents, such as a higher investment from parents or increased income level, but also by some exogenous variables, such as an improvement in education system or the laws on compulsory school leaving age or some political effects. We use a transition matrix to study the social categories of the children given that the categories of their parents are known but do not look at the causal link between education and occupational levels of young and old generations.

In this chapter, we get transition matrix for absolute mobility from raw historical data. From the data we compute the transition matrix and define $a_{IJ}$ as the chance of a child moving from parents in $J$ to $I$ which we measure by $a_{IJ} = n_{IJ}/N_J$, where $n_{IJ}$ is the number of children in social category $I$ whose parents belong to category $J$; $N_J$ is the number of parents in social category $J$. The $a_{IJ}$ is the measure of absolute mobility. Perfect absolute mobility (Labar, 2011) requires $a_{1J} = a_{2J} = \ldots = a_{IJ}$, meaning that a child whose parents are located in category $J$ has an equal chance of being in any group $I$. Complete absolute immobility is defined as $a_{IJ} = 0$ for
A child stays in the same social category as his/her parents and the probability to change his/her social status is zero.

2 Conditional mobility

Unlike absolute mobility, we study conditional mobility through ordered probits to look at the causal link between parental and child’s characteristics. Conditional mobility emphasizes more the effect of parent’ characteristics, such as educational attainment and occupation categories. Time effects, such as economic growth, do not need to be considered in ordered probit model because each individual gets one unique rank relatively to the whole distribution in ordered probit. If the whole distribution is shifted due to economic boom and the rank does not vary, then there will be no time effect in the relative position. In the regressions, we consider the effect of parent’ educational attainment and occupation categories, children’s individual characteristics, macro and idiosyncratic shocks on child’s education attainment and occupation category. We estimate the significance of parental characteristics on child’s mobility. Immobility will be indicated if parental characteristics are significant, meaning that the society is fairly immobile and child stays in the same social category as his/her parent. On the other hand, high mobility holds if parental characteristics are insignificant, meaning that the social category of the young generation is less influenced by that of old generation.

5.4 Data

5.4.1 Data description

The data used in this chapter comes from rural-urban migrant (RUMiCI) in China, 2008. The RUMiCI is an ongoing longitudinal survey that covers the middle and east part of China and was established particularly to investigate the impact of internal migration from less to more developed regions within China. Although it is an ongoing longitudinal survey, we select the 2008 data in this chapter. The 2008 RUMiCI covers the urban, rural and rural-urban migrant individuals and households and includes individual characteristics, individual/household income, household assets/liabilities and household expenditures. For the purpose of our study, we only select rural-urban migrant individual and household data sets (RUH). The 2008 individual data set initially contains 8444 migrants and the household data set initially contains 5007 rural-urban migrant households. The sample sizes are reasonably large. Nevertheless, some problems in the database still remain when we use it for economic topics. One issue (see Tab 9) is that the 2008 data distinguishes between dependent child (pre-school or in-school child) and adult child (who have finished education and entered labor market). Among those two types of children, some of them co-reside with their parents and some do not. We have no information on the income of those children living
away but we do know their educational level and occupational category. To avoid possible selection bias, we merge all children into the household. In addition, we also have no information on the income of grandparents. But this is not a problem for our work as we study permanent income through education and occupation.

We look at IGM among three generations (Fig 51): old generation (grandparent), middle generation (parent) and young generation (child). For grandparent and parent, we split them into male and female. This gives us six different ways to study IGM between them both within gender (e.g. father and son) and cross gender (e.g. father and daughter). For parent and child, due to the small sample size of dependent children, we divide parents into household head and spouse and have not split child by gender. This generates two ways to look at IGM among parent and child.

### 5.4.2 Statistics

Table 10 gives the mean, median, standard deviation, min and max for the principal variables of interest for 2008 data. We compare the educational level and the age gap between those three generations. (i) We see that the male (or female) parent is better educated than his (or her) mother/father and the young child has a higher educational level than the parent. This education profile shows that the educational level tends to increase intergenerationally, probably due to improvements in the educational system. Especially, in Table 10, the average educational level of a male parent (9.50 level equivalent to 11 years schooling) is nearly twice that of his father (5.32 level equivalent to 8 years schooling) and is three times higher than that of his mother (3.87 level equivalent to 6 years schooling). At the mean value, for the middle aged and the old generations, males have marginally higher educational levels than

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63 Noticing that male is still dominating in Chinese household (most of the household heads are male), we have not further distinguished head by gender.

64 His (or her) father/mother refers to the old generation. Here, "his (or her)" means the "male (or female) parent". Male/female parent is the middle aged generation. Adult child stands for the young generation.
females. To conclude, recent generations have better education than the previous generations. We can also see this graphically from Fig 52 and 53 by comparing the educational level between three generations. The young generation is better educated than the middle and old generations. The middle aged generation has better education than the old generation. Males obtain better education than females. The details of educational level can be viewed in the appendix. Further support for this comes from the histogram which reveals a very strong bunching of years of education at the end of a stage level (Fig 54). (ii) The age gap between female parent and her father/mother is generally 30 years, that between male parent and his father/mother is about 25 years and that between adult child and parents is normally 10 years.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Mdn.</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>His father edu</td>
<td>3964</td>
<td>5.32</td>
<td>4.00</td>
<td>3.91</td>
<td>1.00</td>
<td>32.00</td>
</tr>
<tr>
<td>His mother edu</td>
<td>4242</td>
<td>3.87</td>
<td>3.00</td>
<td>3.84</td>
<td>1.00</td>
<td>29.00</td>
</tr>
<tr>
<td>His father age</td>
<td>3630</td>
<td>55.94</td>
<td>54.00</td>
<td>10.48</td>
<td>30.00</td>
<td>96.00</td>
</tr>
<tr>
<td>His mother age</td>
<td>3843</td>
<td>55.05</td>
<td>53.00</td>
<td>11.19</td>
<td>30</td>
<td>108.00</td>
</tr>
<tr>
<td>No of child his mother has</td>
<td>4242</td>
<td>3.05</td>
<td>3.00</td>
<td>1.46</td>
<td>1.00</td>
<td>9.00</td>
</tr>
<tr>
<td>Her father edu</td>
<td>1911</td>
<td>3.69</td>
<td>2.00</td>
<td>3.33</td>
<td>1.00</td>
<td>24.00</td>
</tr>
<tr>
<td>Her mother edu</td>
<td>2110</td>
<td>2.69</td>
<td>2.00</td>
<td>2.57</td>
<td>1.00</td>
<td>24.00</td>
</tr>
<tr>
<td>Her father age</td>
<td>1557</td>
<td>61.45</td>
<td>60.00</td>
<td>9.79</td>
<td>30.00</td>
<td>91.00</td>
</tr>
<tr>
<td>Her mother age</td>
<td>1690</td>
<td>60.74</td>
<td>60.00</td>
<td>10.10</td>
<td>30</td>
<td>97.00</td>
</tr>
<tr>
<td>No of child her mother has</td>
<td>2110</td>
<td>3.59</td>
<td>3.00</td>
<td>1.52</td>
<td>1.00</td>
<td>9.00</td>
</tr>
<tr>
<td>Male parent’s edu</td>
<td>3908</td>
<td>9.50</td>
<td>8.00</td>
<td>4.28</td>
<td>1.00</td>
<td>26.00</td>
</tr>
<tr>
<td>Male parent’s age</td>
<td>3912</td>
<td>32.31</td>
<td>31.00</td>
<td>10.40</td>
<td>15.00</td>
<td>71.00</td>
</tr>
<tr>
<td>Male parent’s duration in urban</td>
<td>3858</td>
<td>8.90</td>
<td>8.00</td>
<td>6.81</td>
<td>0</td>
<td>50.00</td>
</tr>
<tr>
<td>No of child he(married) has</td>
<td>2450</td>
<td>1.39</td>
<td>1.00</td>
<td>0.75</td>
<td>0.00</td>
<td>5.00</td>
</tr>
<tr>
<td>Female parent’s edu</td>
<td>2881</td>
<td>8.62</td>
<td>8.00</td>
<td>4.43</td>
<td>1.00</td>
<td>26.00</td>
</tr>
<tr>
<td>Female parent’s age</td>
<td>2883</td>
<td>31.28</td>
<td>30.00</td>
<td>9.61</td>
<td>15.00</td>
<td>69.00</td>
</tr>
<tr>
<td>Female parent’s duration in urban</td>
<td>2818</td>
<td>7.13</td>
<td>6.00</td>
<td>5.70</td>
<td>0</td>
<td>36.00</td>
</tr>
<tr>
<td>No of child she(married) has</td>
<td>1993</td>
<td>1.34</td>
<td>1.00</td>
<td>0.73</td>
<td>0.00</td>
<td>5.00</td>
</tr>
<tr>
<td>Adult child’s edu</td>
<td>871</td>
<td>10.53</td>
<td>8.00</td>
<td>4.91</td>
<td>1.00</td>
<td>28.00</td>
</tr>
<tr>
<td>Adult child’s age</td>
<td>871</td>
<td>24.37</td>
<td>23.00</td>
<td>6.25</td>
<td>9.00</td>
<td>54.00</td>
</tr>
</tbody>
</table>

Table 10: 2008 Y vs M vs O

Overall, we see that the educational level is increasing and the age gap is decreasing intergenerationally. One interesting aspect in a Chinese context is the social and family network. Most migrants got their job arranged through relatives or friends before migration occurred. This raises the importance of the network especially the size of the networks. The larger the size of the network (the number of family relatives and friends a migrant family normally contacts), the more information on employment a family can get and the higher employment probabilities can be realized for job searching. The size of the network varies a lot among households (Fig 55). Some can have less than 10 but some have nearly 100 relatives and friends but most families have about 10 to 30 people whom they contact frequently. Statistically, we also look at the gender ratio even though the sample size for adult child is small and we have not listed the results here. We find that the male child is mixed with female child in a ratio.
Figure 52: 2008 Edu gap between household head, spouse and adult child

Figure 53: 2008 Edu gap between grandfather, grandmother and middle aged son
of 1:1 in both tables. Nationally, more boys than girls have been born in China since the early 1990s. There is no strong evidence of gender discrimination in rural-urban migrant households. For example, the boys and girls have nearly the same education in 2008.

We have information on the occupational choice of adult children and their parents and grandparents, who are aged between 16 and 55, classified into 25 occupations and three occupation groups (based on the mean value of permanent income, see Tab 8). In Table 11, nearly all grandparents (old generation) are farmers. Most of the rural-urban migrants (middle aged generation) and their adult children (young generation) work in service and manufacturing sectors and only a few of them work in high technical sectors such as the professional sector. Interestingly, 10% of middle aged migrants are self-employed and gain high income. This might be due to the decreasing moving barriers between rural and urban areas and the improving investment environment (such as better network and less restrictions on small business investment) for migrants but also can be due to low job vacancies for migrants. It is worth noticing that a much lower % of the young generation is self-employed only (2.67 %). This might result from financial and credit constraints as the young generation is at the beginning of their lifetime wealth profile. Meanwhile, 9.35% of young generation are rural farmers whilst the majority of the young generation work on urban non-farming sectors. All the young farmers are adult children not living with parents and have intermediate education levels and are mixed with 1:1 in gender. Possibly these adult young farmers were left behind in the rural area when their parents migrated to urban areas and remained to work on the land.

\[65\]\ In one of the groups, we eliminate "family workers and others" occupation as the information on permanent income mostly is missing.
Figure 55: Family and social network

<table>
<thead>
<tr>
<th>Rank</th>
<th>Occupation</th>
<th>Old generation</th>
<th>Old generation</th>
<th>Middle aged</th>
<th>Middle aged</th>
<th>Young generation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>His father</td>
<td>His mother</td>
<td>Parent(Male)</td>
<td>Parent(Female)</td>
<td>Adult-child</td>
</tr>
<tr>
<td>1</td>
<td>Food trades assistants, kitchenhands</td>
<td>0.05</td>
<td>0.05</td>
<td>0.46</td>
<td>0.59</td>
<td>0.45</td>
</tr>
<tr>
<td>2</td>
<td>Cleaners and laundry workers</td>
<td>0.25</td>
<td>0.38</td>
<td>0.11</td>
<td>0.1</td>
<td>1.39</td>
</tr>
<tr>
<td>3</td>
<td>Hospitality workers in restaurant</td>
<td>0.31</td>
<td>0.46</td>
<td>0.21</td>
<td>0.19</td>
<td>5.86</td>
</tr>
<tr>
<td>4</td>
<td>Fruits, veg and rice sales men</td>
<td>0.23</td>
<td>0.1</td>
<td>0.05</td>
<td>2.63</td>
<td>3.91</td>
</tr>
<tr>
<td>5</td>
<td>Recycling and rubbish collectors</td>
<td>0.1</td>
<td>0.05</td>
<td>0.44</td>
<td>0.44</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Protective service workers for factory garage or for local residence (group1)</td>
<td>0.46</td>
<td>0.07</td>
<td>0.11</td>
<td>0.05</td>
<td>14.39</td>
</tr>
<tr>
<td>7</td>
<td>Repair, appliance repairman</td>
<td>0.28</td>
<td>0.02</td>
<td>2.45</td>
<td>0.59</td>
<td>2.23</td>
</tr>
<tr>
<td>8</td>
<td>Personal care assistants, nursing assistants for family</td>
<td>0.13</td>
<td>0.33</td>
<td>0.16</td>
<td>0.24</td>
<td>0.19</td>
</tr>
<tr>
<td>9</td>
<td>Other service workers</td>
<td>0.1</td>
<td>0.05</td>
<td>0.11</td>
<td>0.05</td>
<td>0.36</td>
</tr>
<tr>
<td>10</td>
<td>Handlers and deliveries</td>
<td>0.31</td>
<td>0.02</td>
<td>2.68</td>
<td>0.55</td>
<td>1.1</td>
</tr>
<tr>
<td>11</td>
<td>Maintenance and production services workers</td>
<td>0.03</td>
<td>0.05</td>
<td>2.22</td>
<td>1.07</td>
<td>0.45</td>
</tr>
<tr>
<td>12</td>
<td>Other factory process workers/group2</td>
<td>2.8</td>
<td>1.81</td>
<td>1.8</td>
<td>0.72</td>
<td>4.26</td>
</tr>
<tr>
<td>13</td>
<td>Chefs and barmans</td>
<td>0.18</td>
<td>0.17</td>
<td>5.33</td>
<td>1.37</td>
<td>2.45</td>
</tr>
<tr>
<td>14</td>
<td>Personal service workers (SPA, hairdresser, tourists guide)</td>
<td>2.63</td>
<td>0.05</td>
<td>2.63</td>
<td>3.06</td>
<td>1.56</td>
</tr>
<tr>
<td>15</td>
<td>Sales workers</td>
<td>0.74</td>
<td>0.62</td>
<td>0.32</td>
<td>0.24</td>
<td>8.43</td>
</tr>
<tr>
<td>16</td>
<td>Machinery operators and factory process workers</td>
<td>1.66</td>
<td>1.19</td>
<td>0.85</td>
<td>0.24</td>
<td>10.99</td>
</tr>
<tr>
<td>17</td>
<td>Farmers</td>
<td>78.52</td>
<td>87.42</td>
<td>88.52</td>
<td>94.73</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>Clerical and admin workers</td>
<td>1.3</td>
<td>0.53</td>
<td>0.58</td>
<td>0.24</td>
<td>3.25</td>
</tr>
<tr>
<td>19</td>
<td>Construction labourers</td>
<td>4.31</td>
<td>0.57</td>
<td>1.27</td>
<td>0.29</td>
<td>12.38</td>
</tr>
<tr>
<td>20</td>
<td>Automobile drivers, delivery drivers/group3</td>
<td>0.66</td>
<td>0.07</td>
<td>0.32</td>
<td>2.71</td>
<td>0.33</td>
</tr>
<tr>
<td>21</td>
<td>Technical Professionals</td>
<td>1.73</td>
<td>0.88</td>
<td>1.22</td>
<td>0.88</td>
<td>0.46</td>
</tr>
<tr>
<td>22</td>
<td>Self-employed</td>
<td>2.26</td>
<td>2.34</td>
<td>0.62</td>
<td>0.88</td>
<td>12.06</td>
</tr>
<tr>
<td>23</td>
<td>Managers</td>
<td>0.97</td>
<td>0.17</td>
<td>0.63</td>
<td>0.24</td>
<td>1.62</td>
</tr>
<tr>
<td>24</td>
<td>Owner of private sector/group4</td>
<td>0.87</td>
<td>0.6</td>
<td>0.21</td>
<td>0.14</td>
<td>3.46</td>
</tr>
</tbody>
</table>

Table 11: Occupation group
5.5 Estimation method

- IGM in education

We use the transition matrix to estimate absolute IGM which is derived from observed outcomes. For conditional intergenerational education mobility, we apply an ordered probit model to estimate the effect of parents' educational attainment on children's education level. Education level is a categorical variable. Consequently, econometric methods analysing a continuous random variable are less suitable. In addition, ordered probit model is an ideal framework to study the relative position change between generations. The ordered probit framework has a very tight structure and demands a lot from the data, but it also has a very nice interpretation in terms of the underlying choice process. So we treat \( E_{i,t} \) as an ordered response taking values \( \{1, 2, \ldots, J\} \) for individual \( i \) born at time \( t \), where \( J \) takes 4 values for 2008 survey (see tables in Appendix). The model uses a latent variable,

\[
E^{*}_{i,t} = X_{i,t} \alpha + X_{j,t-1} \lambda + \beta E_{j,t-1} + \varepsilon_t
\]

where \( \varepsilon_t \) is normally distributed; \( X_{i,t} \) is individual characteristics of the child; \( X_{j,t-1} \) is individual characteristics of the parent; \( E_{j,t-1} \) is the educational attainment of parents; \( E^{*}_{i,t} \) is an indicator function for the group to which the adult child belongs. Here individual characteristics include a variety of factors, such as age, social status of the parent and peer effect/network. The peer effect/the family & social network is worth explaining in a Chinese context since the personal network is regarded as a very important factor for education, employment and migration in China. In this chapter, we measure "the number of friends and family relatives" a household has as peer effect/the family & social network. Self-confident and successful households are more likely to contact with a large number of friends, potentially showing a higher value of income. Also, the number of friends and relatives captures the differences in attitude toward education and parenting. Similar arguments can be put forward for the existence of a family & social network, this affects the school choice, quality of friends around and place of residence. A well educated parent with a high level of income might choose to live in an area where the children can get access to better schools, good quality of neighbor and friends and public services. In addition, "the number of friends and family relatives" a household has indicates the size of the information set on education and job vacancies. Having more friends can bring more information on the quality of different universities and different education investment channels to increase efficiency of education investment.

- IGM in occupation

For intergenerational occupation mobility, we estimate the effect of parents' occupation on children's occupation. \( O_{i,t} \) is an ordered response taking values \( \{1, 2, \ldots, I\} \), where \( I \) takes 4 values for 2008 survey. The model is derived
from a latent variable model,

\[ O_{i,t}^* = Z_{i,t} \gamma + \psi Z_{j,t-1} + \lambda E_{i,t} + \eta O_{j,t-1} + u_t \]

where \( u_t \) is normally distributed; \( Z_{i,t} \) is individual characteristics of adult child; \( Z_{j,t-1} \) is the educational attainment of parents; \( O_{i,t} \) is the educational attainment of child; \( O_{j,t-1} \) the occupation level of parent; \( O_{i,t}^* \) is an indicator function for the group to which the individual belongs. Again we start with an ordered probit specification but also keep in reserve the possibility of multinomial logit. If the mean value of permanent income does not capture the natural order of occupation (e.g. the rank of occupation is not only based on permanent income but also on subjective social recognition), then an ordered probit model will not be suitable to study IGM in occupation and multinomial logit will be the better model.

5.6 Results and interpretation

The results for both types of mobility are discussed below.

5.6.1 Absolute mobility

In terms of education level, we find that young and middle aged generations are relatively more educated than the old generation as most of the young and middle aged generations finished junior middle school and some of them even finished senior middle school while the old generation mainly just finished primary school. This can be partly due to general improvement in the education system or due to parental characteristics. By looking at transition matrices (Tab 12), the mobilities within and between genders are not so different. For IGM in occupation (13), it shares the similar features and the young generation tends to have high mobilities at the high occupation group.

To conclude, over time, absolute IGM in education and occupation is relatively high from the bottom to the median level but apart from this, movements are restricted to a small shift up or down and the immobility at the top is high. But the probability for an individual to move from a low category ("elementary" and "junior") to a high category ("high edu") is still low. The IGM in the high category is still low and there is relatively high immobility.

5.6.2 Conditional mobility

To study conditional IGM we want to calculate generational changes which arise after the effects of other causes have been netted out as far as possible. In the background there is a model in section 5 above, where \( E_{i,t}^* \) (or \( O_{i,t}^* \)) is an indicator function for the group to which the individual belongs.

Through the ordered probit models, we explore the determinants of conditional IGM in education using ordered probits with explanatory variables education level of the old generation, age of both generations, peer effect, social
<table>
<thead>
<tr>
<th>Absolute mobility in ed</th>
<th>Middle aged Son/Old Father</th>
<th>1 Elementary</th>
<th>2 Junior</th>
<th>3 Senior</th>
<th>4 High</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Elementary (1 to 6 years schooling)</td>
<td>0.137</td>
<td>0.043</td>
<td>0.035</td>
<td>0.056</td>
<td>0.101</td>
<td></td>
</tr>
<tr>
<td>2 Junior (7 to 9 years schooling)</td>
<td>0.488</td>
<td>0.455</td>
<td>0.389</td>
<td>0.222</td>
<td>0.466</td>
<td></td>
</tr>
<tr>
<td>3 Senior (10 to 12 years schooling)</td>
<td>0.33</td>
<td>0.439</td>
<td>0.472</td>
<td>0.222</td>
<td>0.374</td>
<td></td>
</tr>
<tr>
<td>4 High (13 years schooling above)</td>
<td>0.046</td>
<td>0.062</td>
<td>0.103</td>
<td>0.5</td>
<td>0.06</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Absolute mobility in ed</th>
<th>Middle aged Son/Old Mother</th>
<th>1 Elementary</th>
<th>2 Junior</th>
<th>3 Senior</th>
<th>4 High</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Elementary (1 to 6 years schooling)</td>
<td>0.12</td>
<td>0.021</td>
<td>0.064</td>
<td>0</td>
<td>0.102</td>
<td></td>
</tr>
<tr>
<td>2 Junior (7 to 9 years schooling)</td>
<td>0.472</td>
<td>0.445</td>
<td>0.357</td>
<td>0.333</td>
<td>0.467</td>
<td></td>
</tr>
<tr>
<td>3 Senior (10 to 12 years schooling)</td>
<td>0.361</td>
<td>0.432</td>
<td>0.47</td>
<td>0</td>
<td>0.377</td>
<td></td>
</tr>
<tr>
<td>4 High (13 years schooling above)</td>
<td>0.047</td>
<td>0.102</td>
<td>0.119</td>
<td>0.667</td>
<td>0.06</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Absolute mobility in ed</th>
<th>Middle aged Daughter/Old Father</th>
<th>1 Elementary</th>
<th>2 Junior</th>
<th>3 Senior</th>
<th>4 High</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Elementary (1 to 6 years schooling)</td>
<td>0.133</td>
<td>0.049</td>
<td>0.061</td>
<td>0.083</td>
<td>0.091</td>
<td></td>
</tr>
<tr>
<td>2 Junior (7 to 9 years schooling)</td>
<td>0.53</td>
<td>0.483</td>
<td>0.32</td>
<td>0.25</td>
<td>0.481</td>
<td></td>
</tr>
<tr>
<td>3 Senior (10 to 12 years schooling)</td>
<td>0.289</td>
<td>0.411</td>
<td>0.527</td>
<td>0.5</td>
<td>0.361</td>
<td></td>
</tr>
<tr>
<td>4 High (13 years schooling above)</td>
<td>0.048</td>
<td>0.057</td>
<td>0.143</td>
<td>0.167</td>
<td>0.067</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Absolute mobility in ed</th>
<th>Middle aged Daughter/Old Mother</th>
<th>1 Elementary</th>
<th>2 Junior</th>
<th>3 Senior</th>
<th>4 High</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Elementary (1 to 6 years schooling)</td>
<td>0.144</td>
<td>0.032</td>
<td>0.017</td>
<td>0.143</td>
<td>0.111</td>
<td></td>
</tr>
<tr>
<td>2 Junior (7 to 9 years schooling)</td>
<td>0.494</td>
<td>0.458</td>
<td>0.243</td>
<td>0.429</td>
<td>0.466</td>
<td></td>
</tr>
<tr>
<td>3 Senior (10 to 12 years schooling)</td>
<td>0.312</td>
<td>0.433</td>
<td>0.617</td>
<td>0.289</td>
<td>0.362</td>
<td></td>
</tr>
<tr>
<td>4 High (13 years schooling above)</td>
<td>0.049</td>
<td>0.076</td>
<td>0.122</td>
<td>0.143</td>
<td>0.061</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Absolute mobility in ed</th>
<th>Adult child/Household head</th>
<th>1 Elementary</th>
<th>2 Junior</th>
<th>3 Senior</th>
<th>4 High</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Elementary (1 to 6 years schooling)</td>
<td>0.101</td>
<td>0.063</td>
<td>0.105</td>
<td>0</td>
<td>0.087</td>
<td></td>
</tr>
<tr>
<td>2 Junior (7 to 9 years schooling)</td>
<td>0.566</td>
<td>0.547</td>
<td>0.298</td>
<td>0.3</td>
<td>0.492</td>
<td></td>
</tr>
<tr>
<td>3 Senior (10 to 12 years schooling)</td>
<td>0.273</td>
<td>0.305</td>
<td>0.434</td>
<td>0.5</td>
<td>0.326</td>
<td></td>
</tr>
<tr>
<td>4 High (13 years schooling above)</td>
<td>0.059</td>
<td>0.085</td>
<td>0.162</td>
<td>0.2</td>
<td>0.095</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Absolute mobility in ed</th>
<th>Adult child/Household spouse</th>
<th>1 Elementary</th>
<th>2 Junior</th>
<th>3 Senior</th>
<th>4 High</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Elementary (1 to 6 years schooling)</td>
<td>0.116</td>
<td>0.1043</td>
<td>0.04</td>
<td>0</td>
<td>0.085</td>
<td></td>
</tr>
<tr>
<td>2 Junior (7 to 9 years schooling)</td>
<td>0.534</td>
<td>0.538</td>
<td>0.28</td>
<td>0.25</td>
<td>0.498</td>
<td></td>
</tr>
<tr>
<td>3 Senior (10 to 12 years schooling)</td>
<td>0.278</td>
<td>0.332</td>
<td>0.533</td>
<td>0.5</td>
<td>0.333</td>
<td></td>
</tr>
<tr>
<td>4 High (13 years schooling above)</td>
<td>0.061</td>
<td>0.087</td>
<td>0.147</td>
<td>0.25</td>
<td>0.083</td>
<td></td>
</tr>
</tbody>
</table>

Table 12: Absolute mobility for education
<table>
<thead>
<tr>
<th>Absolute mobility in occup: Middle Aged vs Old</th>
<th>Middle aged Sen/Old Father</th>
<th>1 (low)</th>
<th>2 (middle)</th>
<th>3 (high)</th>
<th>4 (top)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (low)</td>
<td>0.36</td>
<td>0.22</td>
<td>0.27</td>
<td>0.29</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>2 (middle)</td>
<td>0.04</td>
<td>0.18</td>
<td>0.13</td>
<td>0.11</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>3 (high)</td>
<td>0.57</td>
<td>0.51</td>
<td>0.48</td>
<td>0.51</td>
<td>0.49</td>
<td></td>
</tr>
<tr>
<td>4 (top)</td>
<td>0.04</td>
<td>0.12</td>
<td>0.12</td>
<td>0.09</td>
<td>0.12</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Absolute mobility in occup: Middle Aged vs Old</th>
<th>Middle aged Son/Old Mother</th>
<th>1 (low)</th>
<th>2 (middle)</th>
<th>3 (high)</th>
<th>4 (top)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (low)</td>
<td>0.26</td>
<td>0.10</td>
<td>0.27</td>
<td>0.31</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>2 (middle)</td>
<td>0.08</td>
<td>0.10</td>
<td>0.13</td>
<td>0.11</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>3 (high)</td>
<td>0.60</td>
<td>0.69</td>
<td>0.48</td>
<td>0.48</td>
<td>0.49</td>
<td></td>
</tr>
<tr>
<td>4 (top)</td>
<td>0.08</td>
<td>0.04</td>
<td>0.13</td>
<td>0.11</td>
<td>0.12</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Absolute mobility in occup: Middle Aged vs Old</th>
<th>Middle aged Daughter/Old Father</th>
<th>1 (low)</th>
<th>2 (middle)</th>
<th>3 (high)</th>
<th>4 (top)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (low)</td>
<td>0.20</td>
<td>0.13</td>
<td>0.32</td>
<td>0.24</td>
<td>0.31</td>
<td></td>
</tr>
<tr>
<td>2 (middle)</td>
<td>0.00</td>
<td>0.09</td>
<td>0.10</td>
<td>0.06</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>3 (high)</td>
<td>0.40</td>
<td>0.05</td>
<td>0.38</td>
<td>0.35</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>4 (top)</td>
<td>0.40</td>
<td>0.13</td>
<td>0.20</td>
<td>0.35</td>
<td>0.20</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Absolute mobility in occup: Middle Aged vs Old</th>
<th>Middle aged Daughter/Old Mother</th>
<th>1 (low)</th>
<th>2 (middle)</th>
<th>3 (high)</th>
<th>4 (top)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (low)</td>
<td>0.33</td>
<td>0.17</td>
<td>0.31</td>
<td>0.18</td>
<td>0.31</td>
<td></td>
</tr>
<tr>
<td>2 (middle)</td>
<td>0.00</td>
<td>0.08</td>
<td>0.10</td>
<td>0.18</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>3 (high)</td>
<td>0.33</td>
<td>0.67</td>
<td>0.39</td>
<td>0.36</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>4 (top)</td>
<td>0.33</td>
<td>0.08</td>
<td>0.20</td>
<td>0.27</td>
<td>0.20</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Absolute mobility in occup: Adult Child vs Middle aged</th>
<th>Adult child/Middle Aged Householdhead</th>
<th>1 (low)</th>
<th>2 (middle)</th>
<th>3 (high)</th>
<th>4 (top)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (low)</td>
<td>0.32</td>
<td>0.13</td>
<td>0.11</td>
<td>0.11</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>2 (middle)</td>
<td>0.05</td>
<td>0.37</td>
<td>0.11</td>
<td>0.05</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>3 (high)</td>
<td>0.57</td>
<td>0.43</td>
<td>0.67</td>
<td>0.57</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>4 (top)</td>
<td>0.06</td>
<td>0.07</td>
<td>0.11</td>
<td>0.26</td>
<td>0.12</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Absolute mobility in occup: Adult Child vs Middle aged</th>
<th>Adult child/Middle Aged Householdspouse</th>
<th>1 (low)</th>
<th>2 (middle)</th>
<th>3 (high)</th>
<th>4 (top)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (low)</td>
<td>0.34</td>
<td>0.12</td>
<td>0.11</td>
<td>0.04</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>2 (middle)</td>
<td>0.05</td>
<td>0.54</td>
<td>0.09</td>
<td>0.07</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>3 (high)</td>
<td>0.50</td>
<td>0.29</td>
<td>0.76</td>
<td>0.56</td>
<td>0.55</td>
<td></td>
</tr>
<tr>
<td>4 (top)</td>
<td>0.12</td>
<td>0.05</td>
<td>0.04</td>
<td>0.32</td>
<td>0.12</td>
<td></td>
</tr>
</tbody>
</table>

Table 13: Absolute mobility for occupation
status of the older generation. The main results from the ordered probits regression for IGM in education are shown in the tables below. The results for IGM in education show that there is immobility with significant effects of education of the older generation and the middle aged generation. Father’s education level is always more important although the mother’s effect is also significant. This is maybe due to the patriarchal nature of Chinese families. Also, in contrast to the results of absolute mobility, grandmothers have equal effects on son and daughter, but the grandfathers have slightly more impact on daughter’s educational level. Daughters may look to their father as a role model more than sons do. Parental age is important with grouped gender. Interestingly, for sons (male middle aged) and daughters (female middle aged), the number of children the grandmother has and also the number of friends that the household has are significant. The social network seems to be important for education.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient (Std. Err.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>His father edu level</td>
<td>0.173*** (0.037)</td>
</tr>
<tr>
<td>His mother edu level</td>
<td>0.0695 (0.045)</td>
</tr>
<tr>
<td>His father age</td>
<td>0.000569 (0.007)</td>
</tr>
<tr>
<td>His mother age</td>
<td>-0.00489 (0.007)</td>
</tr>
<tr>
<td>No of children of his mother has</td>
<td>-0.0770*** (0.021)</td>
</tr>
<tr>
<td>Age</td>
<td>0.0296 (0.021)</td>
</tr>
<tr>
<td>Agesqu</td>
<td>-0.000929** (0.0003)</td>
</tr>
<tr>
<td>Peer effect (no of friends)</td>
<td>0.00217*** (0.0005)</td>
</tr>
</tbody>
</table>

Table 14: Ordered probit, Son’s (middle aged) edu

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient (Std. Err.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Her father edu level</td>
<td>0.231*** (0.053)</td>
</tr>
<tr>
<td>Her mother edu level</td>
<td>0.0802 (0.058)</td>
</tr>
<tr>
<td>Her father age</td>
<td>-0.0245* (0.010)</td>
</tr>
<tr>
<td>Her mother age</td>
<td>0.0288** (0.011)</td>
</tr>
<tr>
<td>No of children of her mother has</td>
<td>-0.142*** (0.032)</td>
</tr>
<tr>
<td>Age</td>
<td>0.0625 (0.034)</td>
</tr>
<tr>
<td>Agesqu</td>
<td>-0.00150* (0.00058)</td>
</tr>
<tr>
<td>Peer effect (no of friends)</td>
<td>0.00252* (0.001)</td>
</tr>
</tbody>
</table>

Table 15: Ordered probit, Daughter’s (middle aged) edu

For the determinants of conditional occupational mobility (see table below), the occupational group of the household spouse plays an important role on child’s occupation. In addition the education level and the gender of the child determine his/her occupational category. The important role of child’s education in occupational
Table 16: Ordered probit, Adult Child’s (young generation) edu

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>(Std. Err.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head edu level</td>
<td>0.392***</td>
<td>(0.087)</td>
</tr>
<tr>
<td>Spouse edu level</td>
<td>0.171*</td>
<td>(0.081)</td>
</tr>
<tr>
<td>Head age</td>
<td>0.0137*</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Spouse age</td>
<td>0.00544**</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Age</td>
<td>-0.0775*</td>
<td>(0.038)</td>
</tr>
<tr>
<td>Agesqu</td>
<td>0.0000452</td>
<td>(0.0005)</td>
</tr>
<tr>
<td>Gender</td>
<td>0.118</td>
<td>(0.13)</td>
</tr>
<tr>
<td>Peer effect (no of friends)</td>
<td>0.000916</td>
<td>(0.002)</td>
</tr>
</tbody>
</table>

N 305
LR chi 2 (8) 109.91
Prob > chi2 0.0000

destination is clear. Gender plays a negative effect on adult child’s occupational mobility. Boys have low mobility because in an Chinese context, boys are considered as "family heirs" so that they have more chance to inherit parents’ occupation and stay in the same occupation.

Table 17: Ordered probit, Adult Child’s (young generation) Occupation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>(Std. Err.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head occupation category</td>
<td>0.133</td>
<td>(0.1)</td>
</tr>
<tr>
<td>Spouse occupation category</td>
<td>0.375***</td>
<td>(0.1)</td>
</tr>
<tr>
<td>Head age</td>
<td>-0.050</td>
<td>(0.15)</td>
</tr>
<tr>
<td>Head agesqu</td>
<td>0.0005</td>
<td>(0.0015)</td>
</tr>
<tr>
<td>Spouse age</td>
<td>-0.019</td>
<td>(0.06)</td>
</tr>
<tr>
<td>Spouse agesqu</td>
<td>0.0004</td>
<td>(0.0007)</td>
</tr>
<tr>
<td>Edu</td>
<td>0.11**</td>
<td>(0.10)</td>
</tr>
<tr>
<td>Age</td>
<td>0.089</td>
<td>(0.101)</td>
</tr>
<tr>
<td>Agesqu</td>
<td>-0.002</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Gender</td>
<td>-0.46**</td>
<td>(0.18)</td>
</tr>
<tr>
<td>Marital</td>
<td>-0.176</td>
<td>(0.24)</td>
</tr>
<tr>
<td>Employed</td>
<td>-0.007</td>
<td>(0.08)</td>
</tr>
</tbody>
</table>

N 254
LR chi 2 (12) 67.72
Prob > chi2 0.0000

All the ordered probits for education pass the diagnostic test (normality test) and the prediction of the dependent variables matches the sample data well (see tables below). The ordered probits regression for IGM in occupation also generate a well matched prediction but it fails the normality test. The plot (Fig 56) shows pseudo residuals defined as the difference between the actual occupation group and a synthetic group defined by $\sum L_i \hat{p}_{i,L}$, where $L_i = \{1, 2, 3, 4\}$ and $\hat{p}_{i,L}$ is the predicted probability that child $i$ is in level $L$. The plot is asymmetric and skewed with a fat left hand tail. This is far from normality. The skewness and kurtosis suggest use of a Jacques Bera
test: p-values are 0.0061 for Skewness, 0.0436 for Kurtosis and 0.0057 for a joint univariate test. This leads to a rejection of the null hypothesis of normality. For the sake of robustness, we also use multinomial logit regression, which generates similar probabilities to the ordered probit.

Table 18: Son’s (Middle aged) edu (Average predicted percent VS Actual percent)

<table>
<thead>
<tr>
<th>Edu level</th>
<th>Predicted</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elementary (1 to 6 years)</td>
<td>7.80</td>
<td>13.45</td>
</tr>
<tr>
<td>Junior Middle (7 to 9 years)</td>
<td>46.80</td>
<td>45.44</td>
</tr>
<tr>
<td>Senior Middle (10 to 12 years)</td>
<td>38.64</td>
<td>35.93</td>
</tr>
<tr>
<td>Hig edu (above 13 years)</td>
<td>6.74</td>
<td>5.18</td>
</tr>
</tbody>
</table>

Table 19: 2008 Daughter’s (Middle aged) edu (Average predicted percent VS Actual percent)

<table>
<thead>
<tr>
<th>Edu level</th>
<th>Predicted</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elementary (1 to 6 years)</td>
<td>9.11</td>
<td>13.04</td>
</tr>
<tr>
<td>Junior Middle (7 to 9 years)</td>
<td>49.33</td>
<td>46.48</td>
</tr>
<tr>
<td>Senior Middle (10 to 12 years)</td>
<td>35.05</td>
<td>34.62</td>
</tr>
<tr>
<td>Hig edu (above 13 years)</td>
<td>6.50</td>
<td>5.87</td>
</tr>
</tbody>
</table>

Table 20: Adult Child’s (young generation) edu (Average predicted percent VS Actual percent)

<table>
<thead>
<tr>
<th>Edu level</th>
<th>Predicted</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elementary (1 to 6 years)</td>
<td>12.65</td>
<td>12.75</td>
</tr>
<tr>
<td>Junior Middle (7 to 9 years)</td>
<td>37.11</td>
<td>36.60</td>
</tr>
<tr>
<td>Senior Middle (10 to 12 years)</td>
<td>31.49</td>
<td>32.03</td>
</tr>
<tr>
<td>Hig edu (above 13 years)</td>
<td>18.74</td>
<td>18.63</td>
</tr>
</tbody>
</table>

After estimating the regression for the full sample, we compute the four predicted values at mean levels for education/occupation for a given social category $J$ (the category of parental occupation/education). For our own interest, we only did this for young generation. For example the predicted value for occupation group 1 and education group 1 when parents belong to education group 1 and occupation group 1 is displayed below,

$$ pr(I = 1, J = 1 | \hat{E}_E + \hat{\beta}_E E_{j,t-1} + \hat{\alpha}_E X_{i,t} + \hat{\psi}_E X_{j,t-1}) $$

$$ pr(I = 1, J = 1 | \hat{O}_O + \hat{\beta}_O O_{j,t-1} + \hat{\alpha}_O Z_{i,t} + \hat{\psi}_O Z_{j,t-1}) $$

where $\hat{E}_{i,t}/\hat{O}_{i,t}$ is the predicted value; $\hat{\beta}$, $\hat{\alpha}$ and $\hat{\psi}$ with subscripts (E or O) are coefficients for parent education/occupation level $E_{j,t-1}/O_{j,t-1}$, individual characteristics $X_{i,t}/Z_{i,t}$ and $\hat{\psi}$ with subscript the estimated constant term.

We would like to compare the absolute and conditional mobility for the most recent generation to make some possible policy suggestions.
Table 21: Adult Child’s (young generation) occupation (Average predicted percent VS Actual percent)

<table>
<thead>
<tr>
<th>Edu level</th>
<th>Predicted</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>17.86</td>
<td>13.27</td>
</tr>
<tr>
<td>2.</td>
<td>14.97</td>
<td>13.65</td>
</tr>
<tr>
<td>3.</td>
<td>58.94</td>
<td>59.82</td>
</tr>
<tr>
<td>4.</td>
<td>8.23</td>
<td>13.27</td>
</tr>
</tbody>
</table>

For instance, then the conditional transition matrix for occupation (or education) is given by

\[ a_{11} = \Pr(I = 1, J = 1 | c_0 + \beta O_{j,t-1} + \alpha O Z_{i,t} + \psi O Z_{j,t-1}) \] (or, \( a_{11} = \Pr(I = 1, J = 1 | c_E + \beta E_{j,t-1} + \alpha E X_{i,t} + \psi E X_{j,t-1}) \)),

where the occupation (or education) group of parents is fixed at \( J \), the other characteristics of parents, child and economic environment are set at mean level. Through \( a_{IJ} \), we estimate the percentage of children whose predicted group is \( I \) and whose parents are in occupation (or education) category \( J \) (see Table 22 and 23). Compared to the absolute mobility, the advantages of the predicted mobility concept are considering the effect of \( O_{j,t-1} \) (or \( E_{j,t-1} \)) and controlling for the unobserved shocks (\( \varepsilon_{it}/u_{it} \)) on IGM. Comparing the results of absolute transition matrix and predicted transition matrix, generally the results are very similar but the predicted mobility is higher than the absolute mobility at the top and high parts of the education categories. It implies that if all individuals had identical characteristics and environmental variables (at the mean) the chance of moving upwards would increase slightly. This suggests that heterogeneity in the environment can be one cause of the lack of equality of opportunity.

5.6.3 Markov chain predicted dynamics of IGM

Once the absolute or conditional transition matrix has been obtained, we can trace out dynamic movements of the distribution of a variable under the assumption that the transition matrix remains unchanged. We apply this to occupation and education using the absolute transition matrix and then use the link between occupation and permanent income to derive the dynamics of the (permanent) income distribution. There are strong links between occupation and permanent income. Following the same grouping listed above, we find that individuals belong to
lower ranking occupational category have relatively low permanent income, while the 17% individuals in the top occupational category receive the highest permanent income. The monthly permanent income for group 1 is 1127 yuan, for group 2 is 1424 yuan, for group 3 is 1535 yuan and for group 4 is 1995 yuan^67^.

This implies that occupation is a good proxy for lifecycle income and to study the distribution of permanent income.

If $A$ is the transition matrix of interest and at time $t$ the proportion of the population in group $i$ is given by $n_{it}$ (with $n_t$ being the column vector of these proportions) we have

$$n_{t+1} = An_t$$

and iterating this equation, $n_{t+T} = A^T n_t$. Since $A$ is a non-negative matrix with column sums equal to one, as $T$ becomes large $A^T$ usually^68 converges to a limit $A^*$ at which we have long run stationary proportions $n^*$ satisfying $n^* = A^* n^*$. These proportions $n^*$ are shown for the various absolute transition matrices for occupation and education in the table below (Tab 24), using four groups of occupations. We also show the number of generations required for convergence to this long run the distribution to be achieved in the column $T$.

The results indicate that for occupation in 2008 for each group it takes about 10 generations or more for the distribution to converge to a long run position. In the long run there is some concentration of the population in the second highest occupational group, but in other groups there is quite a flat distribution, with very similar

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^67^The income gaps between migrants are not so big but the income gaps between migrants and natives are still big.

^68^In our case, it is convergent.
Table 24: Markov chain proportions in the top and bottom groups. This is true both for intergenerational transmission from the head of household or from the spouse.

The long run distribution for education shows more skewness with the mode in the third group for all cases. The first group is more densely populated than the fourth group for both spouse and head. The long run distribution for education shows similar skewness as that for occupation.

### 5.7 Conclusion

The intergenerational mobility (IGM) between grandparent, parent and child in education and occupation for rural-urban migrants has changed during China’s dynamic evolution but the immobility is still high. We use the absolute transition matrix and ordered probits to capture the effects of parents’ educational attainment and occupational categories on children for 2008. We find that the bottom group is able to help their children to gain social promotion in both educational attainment and occupation. In particular, there are relatively high chances of moving from the bottom to the median educational level but apart from this, movements are restricted to a small shift up or down. There is strong evidence of a time trend of improvement in the level of education and occupation between each pair of generations, matching the economic development of China. For education level, there is a large difference between males and females but this difference falls over time. For occupation, males and females cluster into similar occupation sectors. In terms of IGM for the young generation, the chances of a young generation falling in any education or occupation group differs across groups so that there is evidence of immobility. Next we turn to explore the determinants of IGM in education and occupation using ordered probits with explanatory variables education level and occupation of the old generation, age of both generations and peer effect. The results show that there is immobility with a significant effect of education and occupation of the older generation. For parent and child transition, parental education and occupation are important. In addition, the education level and the marital status of the child determine his/her occupational category.
Treating the transition matrix as a Markov Chain, we find that typically for both variables, after ten or more than ten generations, the distribution has converged to its stationary point. The interpretation is that if there is no policy initiative to shift the intergenerational immobility revealed by the transition matrices, then China will remain with distributions showing inequality and different opportunities for the young depending on their parental background. In addition, the stationary state shows that over time the number of individuals belonging to the bottom part of the distribution has increased, although the mean value of the distribution has increased over time. This implies that if there is no policy initiative, then each individual will benefit from the rising living standards and improving education system but the inequality will be increasing in China.

This links to the idea of equal economic opportunities, which could be achieved when there is complete IGM. Obviously, the empirical evidence suggests that China is still on the way to approaching social justice.

5.8 Some further thoughts

In this chapter, we only looked at the intergenerational mobility as well as the distributions for education and occupation for rural-urban migrants. This can give us an incomplete picture of the changes in the intergenerational mobility over time and the link between mobility and inequality in China. As we know, rural-urban migrants act as a minority group who may receive discrimination in education and occupation choices. Migrants can be pushed to study or work in a separate market facing limited choices such as receiving low quality of education and working in a low paid sector. Because of the Hukou system in China, migrants may be not allowed to work in the same sectors as the urban residents. This may lead to economic immobility for migrants. To better understand immobility in urban area in China, we will include urban residents to compare the mobility levels between urban people and migrants. By controlling the differences in education group and individual characteristics, if urban residents have higher mobility than migrants, there might be some discrimination against migrants or barriers in urban area. There is a need for policy innovation to improve economic situation for migrants. Besides, we want to know whether migration is really a useful channel to change mobility in a society. We will include rural residents in our empirical study to compare the mobility between migrants and non-migrants.

Further, we can have a geographical comparison in mobility both migrants and natives between developed and undeveloped regions in China. For example, we can study the mobility in a spatial context: the coast areas, the middle regions and the west undeveloped places in China. The same type of migrants between coast areas and west regions might face different levels of mobility because of different economic environment (e.g. economic growth rate, or economic structure or discrimination against migrants). Results here would suggest a basis for regional economic development policies to reduce regional differences in equality of opportunity. For example a regionally
based inheritance tax or regional tax on houses purchased and then donated to children.

All these extensions will generate a more complete image and deepen our understanding in mobility in China.
6 Final conclusion

6.1 Conclusion

Regarding the research questions in chapter 1, we used four chapters to try to provide some answers through the theoretical frameworks and empirical applications.

- Chapter 2

In chapter 2, we learned that migration is an important element for economic growth and economic inequality among regions. In a market with a flexible wage rate, migration flow acting as a factor from the labour supply side influences the market wage rate. When there is a substantial amount of migrants moving to a region, the wage rate in this region might fall because of excess supply. Meanwhile, migration flow affects employment probability when there is excess labour supply. The amount of jobs provided is less than that of jobs required by job seekers and then the employment probability decreases. However, when there is excess labour demand, then the employment probability would still be high. Hence, labour demand is also important for migration flows. In this chapter, we studied the labour demand in a representative firm’s profit maximisation process with labour and capital inputs. The labour demand can be increased when there is a high level of capital input and capital does not substitute labour. The employment probability would still be high. Therefore, we introduced the capital market in the analysis and studied the interactions between the labour and capital markets. The migration flow affects the market wage rate and employment probability. On the other hand, migration flow is also determined by the wage rate and employment probability, which are set by labour demand and supply. This requires a dynamic link both labour market and capital as well. Therefore, it is important to study the migration process in a dynamic setting allowing for the interactions between capital and labour markets.

For the dynamic setting, the employment is determined by a real wage rate and fixed capital input in the representative firm’s profit maximization process. Net migration is then determined by the expected income gap between regions. The real wage adjusts according to the partial equilibrium of labor demand and labor supply. The capital (FDI) adjusts dynamically following the time path of the marginal product of capital in country $i$ and the world interest rate at time $t$. Our migration theory not only includes the determinants of one way migration (immigration) found in the existing literature, but also considers two way migration (immigration and return migration) and labour immobility arising from the migration cost. Importantly, our chapter is the first theoretically to recognize that the moving cost causes inherent regime shifts between immigration, return migration and immobility in the dynamic migration process. The non-smoothness and non-differentiable properties of solution
paths mean that we extend the standard methods of local stability analysis to take account of these special features. This approach should have wider applicability than the present context; in particular any market with a similar switching cost should exhibit the same possibility of immobility and regime shift, thus necessitating the local stability methods used here. The elasticity of labor demand plays an important role in immigrants’ extensive margin decisions, in the global dynamics and in the local stability conditions. Stationary states for the three dimensional system exist when the capital plane intersects with the employment plane within the immobility region. When they do exist, there is an infinite number of stationary states lying in the immobility region and along the intersection of the equilibria in the labour and capital markets.

For our empirical work, regions with high levels of capital stock and high real urban wage rates (the Pearl River delta cities especially city 1 and 2) normally have a high number of net migrants. While the labor intensive and low urban real wage rate cities (the northern mountain and the southern agricultural regions) have a small number of net migrants. So the regions in Guangdong exhibit positive dynamic links between the three endogenous variables and regions in the same cluster have similar links. Also the dynamic adjustment speeds in different variables are positively related. The places with high capital flow adjustment speeds have high values for migration adjustment and relatively fast real wage adjustment. The backward places with slow capital adjustment speeds normally show low values for the adjustment speeds of migration and real wage. Compared with the adjustment speeds for capital, the overall relatively small value for the real wage and the migration adjustment speeds in Guangdong can be explained by institutional barriers such as the hukou system and government interventions. These suggest that Guangdong is still in the transitional process to a market system and moving barriers especially the hukou system constrain labour mobility. Also we find that the capital market in Guangdong has not been negatively affected by the 1997 Asian financial crisis. This result is consistent with current results supported by the research on the impact of FDI in China (Pan, 2003).

Overall, we find that migration flow is dynamically and positively linked to the real wage rate and capital flow. The regions with relatively higher real wage rate and a high level of capital stock tend to have a high volume of net migrants. The regions with relatively high adjustments for real wage rate and capital stock tend to have a high adjustment speed for migration flow. Migration is an important element for economic growth and economic inequalities among regions. But the overall adjustment adjustment speed for migration is small. We interpret this as due to high moving cost. Migration cannot overcome or eliminate economic inequality when the moving cost is too high.

- Chapter 3
In chapter 3, we extended one destination to multiple destinations and one moving factor to multiple factors to study the importance of migration on regional disparities within a micro-structured framework, abstracting from temporary migration, planned reverse migration and commuting/guest working. We develop such a multi-motive and multi-location theory to determine the aggregate net migration flows between locations. We assume that all individuals will agree on a ranking of locations from best to worst. Those individuals with bad experiences in their current location will gain the most by moving to the location which is universally judged the best. Individuals who have current utility above the average for their present location may prefer to remain in their current location especially if the migration cost is substantial. So less attractive locations will have emigration especially of the lowest utility inhabitants while the best location will have inward movement. Individuals who start off in the best location, but whose individual experience in that location is much below the location average may find it advantageous to move into the second most attractive location. So on balance the second best location may have net immigration or net emigration and the best location has net immigration. Other locations have net emigration.

We apply the theory to 18 city areas of Guangdong, a east region with the highest volume of internal migration in China. We use a city level panel data over 1990-1998 to econometrically investigate net migration flows between the cities allowing for cross section heteroscedasticity. We start the estimation from a very general model containing all the variables for all cities. Through $T$-tests, $F$-tests, cross section dependence and serial correlation tests and some other diagnostic tests listed in the chapter, we are then be able to nest the general model to a simple model containing just 15 specific coefficients. Nearly half of the cities share a common mean amount of net migration which is unrelated to the four differentials we identified.

In a locational equilibrium, we also looked at the changes (the coefficient of variation of key variables studied in this chapter: wage gap, capital per person etc.) in inequalities between cities over time, which have been increasing not falling over our sample. The rising inequality in some migration inducing factors may imply that a full locational equilibrium has not yet been achieved. We interpret the persistent inequality as due to moving barriers such as Hukou among regions in Guandong, China. This interpretation is consistent with the numerical simulation result found in Whalley and Zhang’s paper (2007).

- Chapter 4

In chapter 4, we studied how to overcome economic inequality through education and transfers between generations using a self-sustained system. We want to understand whether the goal can be achieved in a private system without help from public intervention. Therefore, we developed analytic frameworks within household decision theory to understand intergenerational transfers and education investment in an altruistic-OLG setting with three
generations. We consider a non-cooperative household environment, in which three generations (the O, the M and the Y) live in the same household and only the middle aged generation is an independent decision maker and altruistic. However, each individual lives three periods so the roles of being a decision maker and altruism change over time. We assume that the uneducated Y and retired O do not work and the M is the only working generation. The income of the M is the main financial resource for the whole family, the O relies on the gift sent by the M and the amount of good stored (or the amount of money left in the pot) in the previous period. The intergenerational transfers are normally in the forms of gifts and training investment and the amount of the transfers vary with the degree of altruism. For the incentives to do transfers, we start from a mixture of censored altruism (warm glow/social expectation) and semi-altruism (bounded rationality), and then move to the pure altruism (pure love) to study whether the private Nash equilibrium in intertemporal transfers can be a social optimum. Generally, we find that the social optimum cannot be achieved through the private NE. In order to correct this, we move to pure altruism and turn for help to taxation/subsidy.

- Chapter 5

In chapter 5, we empirically studied the link between migration and economic inequality in lifecycle income (proxied by education and occupation) between generations by comparing the intergenerational mobility (IGM) between migrants’ parents, migrants and migrants’ children. We interpret the increasing mobility across migrant generations as empirical evidence to support an increasing mobility over time between generations. We then further investigate whether there is a positive link between migration and economic inequality over time.

We find that mobility is increasing among generations. There is strong evidence of a time trend of improvement in the level of education and occupation between each pair of generations, matching our hypothesis and the economic development of China. But there is a large difference between males and females even though this difference falls over time. We then use the absolute transition matrix and ordered probits to capture the effects of parents’ educational attainment and occupational categories on children for 2008. There are relatively high chances of moving from the bottom to the median educational level but apart from this, movements are restricted to a small shift up or down. The results show that there is immobility with a significant effect of education and occupation of the older generation in China. The mobility level has increased slightly over time.

Treating the transition matrix as a Markov Chain, we study the distributions of education and occupation to see whether the distributions have converged to its stationary point and how long do they take to converge. We find that typically for both variables, after ten or more than ten generations, the distribution has converged to its stationary distribution. The interpretation is that if there is no policy initiative to shift the intergenerational immobility
revealed by the transition matrices, then China will remain with distributions showing inequality and different opportunities for the young depending on their parental background. In addition, the stationary state shows that over time the number of individuals belonging to the bottom part of the distribution has increased, although the mean value of the distribution has increased over time. This implies that if there is no policy initiative, then each individual will benefit from the rising living standards and improving education system but the inequality will be increasing in China.

6.2 Future research

- Chapter 2

The novelties of our theoretical and empirical work suggest some further studies. We only consider the dynamic interactions between labour and capital markets in the destination. We could think about the dynamics in origin as well. As we allow return migration in our framework, the wage rate in origin should be influenced by return migration as well. If the migrants bring some wealth and send remittances back, the capital market in origin will receive positive inflow. More jobs can be created. People may get job in the origin and stay there rather than migrating to the destination.

Also, theoretically we applied a market adjustment model with a variable wage rate, investment flows and migration. In our model, we have a mechanism for determining not only equilibrium price (wage rate) but also equilibrium quantity (migration and capital flows). The dynamic model developed in this chapter has a disequilibrium process in quantity and price. The structure of the equations is dictated by economic theory. This idea is different from that of stochastic time series models such as VAR (vector autoregression) models and ECM (error correction model). In these the time dependence within and between a list of variables is determined, and if they are cointegrated, their behaviour through time is composed of a moving long run position combined with short term adjustment to deviations from the long run relationships. The VAR and ECM type of models normally just require a list of variables which can be hypothesized to affect each other intertemporally and impose no prior structure. After determining the empirical time series properties of the variables, restrictions on coefficients can be imposed but the specification starts from the data not from economic theory. What is the difference between our model and stochastic time series models? Is there any link? What can the real world or empirical evidence tell us? Does the real work really behave as a market economy or a stochastic process? All these preliminary questions open our thoughts in our future research agenda.

- Chapter 3
We could allow the wage rate to adjust in each location according to labour demand and supply and add non-random matching frictions (or searching frictions) for migrants. From the supply side, migrants are still utility maximizers who are seeking for the highest net return of relocation based on the gain they could get in the host places, the benefit they need to forego in the origin and the matching friction. The matching friction can be either due to characteristics (e.g. low skill to search for a job), or asymmetric information, or market discrimination against migrants, or high level of moving costs, or big entry barriers (e.g. hukou in China). All these exogenous factors cause non-random matching frictions. From the demand side, firms are profit maximizers who employ both migrants and natives. If there is a high level of discrimination against migrants in the local market, then firms will employ more natives than migrants. Otherwise, migrants and natives face equal opportunities for the same job. The current wage rate is endogenously determined by the supply and demand. The market equilibrium is achieved when the supply equals demand.

Another direction for extension, along the line of the one sided matching would be to consider household rather than individual migration in the matching process. E.g. in a two adult household with one dependent child, the migration choice will be made based when the gain of reallocation in destination exceeds the overall household utility in origin.

Due to micro data limitations, we made some strong assumptions in the theoretical framework such as agreement on city ranking among individuals. Now we have some rich microdata sets at hand, we could relax those assumptions and apply it to the microdata sets.

- Chapter 4

This chapter only explores the link between the intergenerationally altruistic transfers and education investment. Some other aspects could be investigated in the future. There could be different types of children: well-behaved children and naughty children. Parents may have asymmetric information on children’ type. The well-behaved children would give true information to their parents and give repayment when their parents get old. The naughty children would report false information to their parents and may not give repayment when their parents get old. We could have a family loan with rotten kids story: within this three period environment, parents could lend money to their kids in the first period and the kids pay back the loan in the second period when their parents become old. In this story, we could bring pooling strategy, or IC (incentive compatibility constraint) or punishment into the modelling part to understand what is the best response if the young generation is not committed or if the parent does not have complete information on the children. (i) One possible approach that the parent can use is a pooling strategy, making all child types do the same amount of repayment. But this strategy leads to a low education
investment because parents would like to get a secured repayment which is a low amount of repayment made by the naughty children. Knowing this, parents would make a low education investment. The well-behaved children would get a low amount of education investment as well. This pooling strategy is not efficient as the well-behaved children cannot get a high education investment, may leading to some welfare loss. (ii) We can also set an appropriate incentive compatibility constraint to prevent naughty children from cheating (e.g. Browning, 2009). (iii) We can set up a punishment, such as a big financial fine or social reputation. A heavy financial fine will be levied so long as the children cheat. All these are just preliminary thoughts, which need to be carefully designed when we do the modelling.

Empirically, there are also a few possible research topics. For instance, (i) we could test for altruistic behavior using a lab experiment (e.g. Andreoni et al, 2007); (ii) we could test the importance of love (e.g. time spent on child care) and money (e.g. education investment) on the outcome of children’s HC and investigate whether these two could be substitute or complementary. The outcome of the HC might be different between the child who was sent to the private school but not receiving enough child care and the child who was sent to public school but receiving enough child care.

- Chapter 5

To better understand immobility in urban area in China, we will include urban residents to compare the mobility levels between urban people and migrants. By controlling the differences in education group and individual characteristics, if urban residents have higher mobility than migrants, there might be some discrimination against migrants or barriers in urban area. There is a need for policy innovation to improve economic situation for migrants. Besides, we want to know whether migration is really a useful channel to change mobility in a society. We will include rural residents in our empirical study to compare the mobility between migrants and non-migrants.

Further, we can have a geographical comparison in mobility between developed and undeveloped regions in China. For example, we can study the mobility in a spatial context: the coast areas, the middle regions and the west undeveloped places in China. The same type of migrants between coast areas and west regions might face different levels of mobility because of different economic environment (e.g. economic growth rate, or economic structure or discrimination against migrants). Regarding different economic environment, different policy can be innovated. All these extensions will generate a more complete image for mobility and deepen our understanding in mobility in China.

- The Next Major Stage: Micro data sets, Migration, Education and Economic development in the Destination
As we studied in this thesis, migration and education are two important tools to assist economic development, improving individual well-being and helping reduce economic inequality to some extent. Our empirical work in Chapters 2 and 3 has used aggregate migration flows to capture its effects, the equations used built up from micro modelling but the data was aggregate. My next aim is to develop and apply microstructure models with a dynamic (multiperiod) dimension to microdata sets on the destination and origin locations.

Overall, we hope to develop two theoretical frameworks for two projects and to apply them empirically using data from the following list: the SLI (Swedish Longitudinal Immigrant Database over 1968-2001), RUMIC (Rural-Urban Migration in China for year 2002, 2008-2010), BHPS (British Household Survey over 1991-2008), Understanding Society over 2010-2011, NSDD (National Survey of Demographic Dynamics 1997) and MNOES (Mexican National Occupation and Employment Survey over 2008-2009) to understand the effect of migration/education on reducing poverty in both destinations and origins.

**Project 1**

For project 1, we hope to develop a sequential choice model to study a dynamic assimilation process in education and occupation of the 1st and 2nd generation immigrants in destination using SLI, RUMIC, MNOSES and BHPS. Each migrant faces different choices in education and occupation (wage worker or self-employed). In pre-migration, individuals are divided into two groups based on their characteristics (education level, working experience, wealth, assets and the size of the network). Both groups face a moving cost. Individuals can make their educational and occupational choices in the first period. Through one period of education or working in the destination, individuals are allowed to make choices on education and occupation again in the second period. Using backward induction, individuals make their choices on education and occupation in a two-stage game setting to maximise their utilities. Some educated migrants might assimilate with natives. Others cannot to do so and get stuck at the initial stage because of liquidity constraints or some other reasons. The research questions we interested in are

1. For which type of individuals and from which origins, is migration a feasible choice?
2. What individual or infrastructure variables determine the choices, assimilation and success of migrants in the destination?
3. What policies will raise the success rate of migrants in the destination?
After the theoretical framework, some policy instruments can be simulated to help people assimilate into the destination, such as lowering the moving cost directly (e.g. visa policy) or expanding the range of choices available in the destination (e.g. poor migrants may have insufficient assets to finance education or to start a business, their only feasible choice is to try to enter the labour market; educational subsidy or scholarship or subsidised business startup finance can overcome this) or increasing the probabilities of securing a wage job (e.g. job searching agencies; job creation; free language tuition).

In the end, a discrete choice utility based framework for occupation choice will be empirically estimated by using MLE.

**Project 2**

For project 2, we hope to develop a dynamic framework within an OLG setting and with altruism to study the effect of migration/remittance on the young generation in the origins. Especially, we are interested in the impacts of migration/remittances on education. The research question we interested in are

1. Remittances/migration as channels for investment, does it have brain drain or gain effect on the young generation in the origins? Do households with migrants/remittances tend to have higher investment on education than the households without? What are the effects of migrants/remittances on different productivity households?

2. Besides remittances/migration, what are the other factors (e.g. socioeconomic and political shocks and environment, non-pecuniary elements) influence human capital?

3. Is there an ideal policy to change the impacts?

4. By applying the theoretical framework below empirically to different countries such as China and Mexico, what are the differences in empirical results?

Similarly, after the theoretical framework, some policy instruments can be simulated to help people develop in the origin, such as tax or subsidy. We could consider a government subsidy on education and tax deductibility of remittances against personal income tax.

From the data, we hope to estimate the reduced form of education investment decisions conditional on the migration decisions of the middle aged.
## Description of key variables

### Wu

<table>
<thead>
<tr>
<th>Formula</th>
<th>Description</th>
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<tbody>
<tr>
<td>( w_{u,t} = \frac{\text{average wage index in top three units}<em>{i,t}}{\text{CPI index}</em>{i,t}} )</td>
<td>( \text{where, average wage index in top three units}<em>{i,t} = \frac{\text{average wage in top three units}</em>{i,t}}{\text{average wage in top three units}_{i,t}} )</td>
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<tr>
<td>( w_{u} = \sum_{t=0}^{9} w_{u,t} \times (*) )</td>
<td>( i = 1...18, t = 0...9 )</td>
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<tbody>
<tr>
<td>( w_{r,t} = \frac{\text{average gross rural income index}<em>{i,t}}{\text{CPI index}</em>{i,t}} )</td>
<td>( \text{where, average gross rural income}<em>{i,t} = \frac{\text{average gross rural income}</em>{i,t}}{\text{average gross rural income}_{i,t}} )</td>
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<tr>
<td>( w_{r} = \sum_{t=0}^{9} w_{r,t} \times (*) )</td>
<td>( i = 1...18, t = 0...9 )</td>
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### UH rate

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<tr>
<td>( \text{urban hukou rate}<em>{i,t} = \frac{\text{urban hukou population}</em>{i,t}}{\text{city population}_{i,t}} )</td>
<td>( \text{urban hukou rate}<em>{i,t} = \sum</em>{t=0}^{9} \text{urban hukou rate}_{i,t} \times (*) )</td>
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<tr>
<td>( i = 1...18, t = 0...9 )</td>
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### Lmar-rate

Late marriage rate \( r_{i,t} \) is the ratio of the number of females who were at least 23 years old at marriage to the total number of first marriages.

### K/P

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<tbody>
<tr>
<td>( (K/P)<em>{i,t} = \frac{\text{capital stock}</em>{i,t}}{\text{city population}_{i,t}} )</td>
<td>( \text{capital stock}<em>{i,t} = K</em>{i,t} - (1 - \delta)K_{i,t-1} + FDI_{i,t} )</td>
</tr>
<tr>
<td>( \delta = \frac{\text{original}K_{i,1992} - \text{net}K_{i,1992}}{\text{original}K_{i,1992}} )</td>
<td>( \delta ) is depreciation rate.</td>
</tr>
<tr>
<td>( i = 1...18, t = 0...9 )</td>
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### SFP

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<tr>
<td>( \text{number of single female}_{i,t} = )</td>
<td>( \text{number of female}<em>{i,t} - \text{number of married and child bearing age}</em>{i,t} )</td>
</tr>
<tr>
<td>( i = 1...18, t = 0...9 )</td>
<td>( \text{where, child bearing age is between 20-49 years old} )</td>
</tr>
<tr>
<td>Description of explanatory variables</td>
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</tr>
<tr>
<td>NM</td>
<td>Number of net migrants</td>
</tr>
<tr>
<td><strong>Ewu-gap</strong></td>
<td>If $i=1(x)$, expected urban wage $1(x) = \sum \text{urban hukou}_j$</td>
</tr>
<tr>
<td></td>
<td>$-$ expected urban wage $2(x) \times \text{urban hukou}_1(x)$</td>
</tr>
<tr>
<td></td>
<td>If $i=2(x)$, expected urban wage $2(x) \times \text{urban hukou}_1(x)$</td>
</tr>
<tr>
<td></td>
<td>$-$ expected urban wage $1(x) \times \text{urban hukou}_2(x)$</td>
</tr>
<tr>
<td></td>
<td>If $i=3(x)$, $-$expected urban wage $1(x) \times \text{urban hukou}_3(x)$;</td>
</tr>
<tr>
<td></td>
<td>$j=1...18, i \neq j$</td>
</tr>
<tr>
<td><strong>P-gap</strong></td>
<td>If $i=1(x)$, city population $1(x) = \sum \text{agricultural hukou}_j$</td>
</tr>
<tr>
<td></td>
<td>$-$city population $2(x) \times \text{agricultural hukou}_1(x)$</td>
</tr>
<tr>
<td></td>
<td>If $i=2(x)$, city population $2(x) \times \text{agricultural hukou}_1(x)$</td>
</tr>
<tr>
<td></td>
<td>$-$city population $1(x) \times \text{agricultural hukou}_2(x)$</td>
</tr>
<tr>
<td></td>
<td>If $i=3(x)$, $-$city population $1(x) \times \text{agricultural hukou}_3(x)$;</td>
</tr>
<tr>
<td></td>
<td>$j=1...18, i \neq j$</td>
</tr>
<tr>
<td><strong>K-gap</strong></td>
<td>If $i=1(x)$, capital stock $1(x) = \sum \text{agricultural hukou}_j$</td>
</tr>
<tr>
<td></td>
<td>$-$capital stock $2(x) \times \text{agricultural hukou}_1(x)$</td>
</tr>
<tr>
<td></td>
<td>If $i=2(x)$, capital stock $2(x) \times \text{agricultural hukou}_1(x)$</td>
</tr>
<tr>
<td></td>
<td>$-$capital stock $1(x) \times \text{agricultural hukou}_2(x)$</td>
</tr>
<tr>
<td></td>
<td>If $i=3(x)$, $-$capital stock $1(x) \times \text{agricultural hukou}_3(x)$;</td>
</tr>
<tr>
<td></td>
<td>$j=1...18, i \neq j$</td>
</tr>
<tr>
<td><strong>SFP-gap</strong></td>
<td>If $i=1(x)$, single female $1(x) = \sum \text{single female}_j$</td>
</tr>
<tr>
<td></td>
<td>$-$single female $2(x) \times \text{single female}_1(x)$</td>
</tr>
<tr>
<td></td>
<td>If $i=2(x)$, single female $2(x) \times \text{single female}_1(x)$</td>
</tr>
<tr>
<td></td>
<td>$-$capital stock $1(x) \times \text{single female}_2(x)$</td>
</tr>
<tr>
<td></td>
<td>If $i=3(x)$, $-$single female $1(x) \times \text{single female}_3(x)$;</td>
</tr>
<tr>
<td></td>
<td>$j=1...18, i \neq j$</td>
</tr>
<tr>
<td><strong>UH-gap</strong></td>
<td>If $i=1(x)$, urban hukou $1(x) = \sum \text{agricultural hukou}_j$</td>
</tr>
<tr>
<td></td>
<td>$-$urban hukou $2(x) \times \text{agricultural hukou}_1(x)$</td>
</tr>
<tr>
<td></td>
<td>If $i=2(x)$, urban hukou $2(x) \times \text{agricultural hukou}_1(x)$</td>
</tr>
<tr>
<td></td>
<td>$-$urban hukou $1(x) \times \text{agricultural hukou}_2(x)$</td>
</tr>
<tr>
<td></td>
<td>If $i=3(x)$, $-$urban hukou $1(x) \times \text{agricultural hukou}_3(x)$;</td>
</tr>
<tr>
<td></td>
<td>$j=1...18, i \neq j$</td>
</tr>
</tbody>
</table>
Appendices for chapter 4

Inter-temporal problem for consumption and gift & bequest

Mathematically, the inter-temporal PO for consumption \((C_{t,t}, C_{t+1,t}, C_{t+2,t})\) is equivalent to that for intra-transfer \((R_{t+1,t}, B_{t+2,t})\). Given that \(c = C_t(B_{t,t-2}, C_{t+1,t}, C_{t+2}(B_{t+2,t}))\) at time \(t\).

Let say, the maximization problem is written as

\[
\max f(C_{t,t}, C_{t+1,t}, C_{t+2})
\]
\[s.t.: \quad g(C_{t,t}, C_{t+1,t}, C_{t+2}) = 0
\]

This maximization problem can be solved in two ways:

\[
L_1 = f(C_{t,t}, C_{t+1,t}, C_{t+2}) + \lambda g(C_{t,t}, C_{t+1,t}, C_{t+2})
\]
\[
\Rightarrow \begin{cases}
\frac{\partial L_1}{\partial C_{t,t}} = \frac{\partial f}{\partial C_{t,t}} + \lambda \frac{\partial g}{\partial C_{t,t}} \\
\frac{\partial L_1}{\partial C_{t+1,t}} = \frac{\partial f}{\partial C_{t+1,t}} + \lambda \frac{\partial g}{\partial C_{t+1,t}} \\
\frac{\partial L_1}{\partial C_{t+2,t}} = \frac{\partial f}{\partial C_{t+2,t}} + \lambda \frac{\partial g}{\partial C_{t+2,t}}
\end{cases} \quad (*)
\]

\[
L_2 = f(C_{t,t}(B_{t,t-2}), C_{t+1,t}(B_{t+1,t}), C_{t+2}(B_{t+2,t}))
\]
\[
+ \lambda g(C_{t,t}(B_{t,t-2}), C_{t+1,t}(B_{t+1,t}), C_{t+2}(B_{t+2,t}))
\]
\[
\Rightarrow \begin{cases}
\frac{\partial L_2}{\partial B_{t,t-2}} = \frac{\partial f}{\partial C_{t,t}} + \lambda \frac{\partial g}{\partial C_{t,t}} \\
\frac{\partial L_2}{\partial B_{t+1,t}} = \frac{\partial f}{\partial C_{t+1,t}} + \lambda \frac{\partial g}{\partial C_{t+1,t}} \\
\frac{\partial L_2}{\partial B_{t+2,t}} = \frac{\partial f}{\partial C_{t+2,t}} + \lambda \frac{\partial g}{\partial C_{t+2,t}}
\end{cases} \quad (**)
\]

If vector
\[
\begin{bmatrix}
\frac{\partial C_{t,t}}{\partial C_{t+1,t}} \\
\frac{\partial C_{t,t}}{\partial C_{t+2,t}} \\
\frac{\partial C_{t+1,t}}{\partial B_{t+1,t}} \\
\frac{\partial C_{t+1,t}}{\partial B_{t+2,t}} \\
\frac{\partial C_{t+2,t}}{\partial B_{t+1,t}} \\
\frac{\partial C_{t+2,t}}{\partial B_{t+2,t}}
\end{bmatrix}
\]
is nonsingular and
\[
\begin{bmatrix}
\frac{\partial f}{\partial C_{t,t}} + \lambda \frac{\partial g}{\partial C_{t,t}} \\
\frac{\partial f}{\partial C_{t+1,t}} + \lambda \frac{\partial g}{\partial C_{t+1,t}} \\
\frac{\partial f}{\partial C_{t+2,t}} + \lambda \frac{\partial g}{\partial C_{t+2,t}}
\end{bmatrix} = 0,
\]
then problem (*) is equivalent to problem (**).

In this chapter, we know that \(C_{t+1,t} = w_{t+1} R_{t+1,t} - R_{t+1,t} , \) clearly
\(C_{t+2} = R_{t+2,t} - B_{t+2,t} \) is non-singular. And
\[
\begin{bmatrix}
\frac{\partial f}{\partial C_{t,t}} + \lambda \frac{\partial g}{\partial C_{t,t}} \\
\frac{\partial f}{\partial C_{t+1,t}} + \lambda \frac{\partial g}{\partial C_{t+1,t}} \\
\frac{\partial f}{\partial C_{t+2,t}} + \lambda \frac{\partial g}{\partial C_{t+2,t}}
\end{bmatrix} = 0
\]
can be easily satisfied according to PO section. Thus, the inter-temporal PO for consumption \((C_{t,t}, C_{t+1,t}, C_{t+2,t})\) is equivalent to that for intra-transfer \((R_{t+1,t}, B_{t+2,t})\).
## Appendices for chapter 5

### Educational category

### Table 25: Education level and category for ordered probit

<table>
<thead>
<tr>
<th>No</th>
<th>Edu level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Five-year elementary (graduate)</td>
</tr>
<tr>
<td>2</td>
<td>Five-year elementary (leave before graduate)</td>
</tr>
<tr>
<td>3</td>
<td>Six-year elementary (graduate)</td>
</tr>
<tr>
<td>4</td>
<td>Six-year elementary (leave before graduate)</td>
</tr>
<tr>
<td>5</td>
<td>two-year junior middle school (graduate)</td>
</tr>
<tr>
<td>6</td>
<td>two-year junior middle school (leave before graduate)</td>
</tr>
<tr>
<td>7</td>
<td>three-year junior middle school (graduate)</td>
</tr>
<tr>
<td>8</td>
<td>three-year junior middle school (leave before graduate)</td>
</tr>
<tr>
<td>9</td>
<td>two-year senior school (graduate)</td>
</tr>
<tr>
<td>10</td>
<td>two-year senior school (leave before graduate)</td>
</tr>
<tr>
<td>11</td>
<td>three-year senior school (graduate)</td>
</tr>
<tr>
<td>12</td>
<td>three-year senior school (leave before graduate)</td>
</tr>
<tr>
<td>13</td>
<td>vocational senior secondary school (graduate)</td>
</tr>
<tr>
<td>14</td>
<td>vocational senior secondary school (leave before graduate)</td>
</tr>
<tr>
<td>15</td>
<td>small secondary school (after junior middle school) (graduate)</td>
</tr>
<tr>
<td>16</td>
<td>small secondary school (after junior middle school) (leave before graduate)</td>
</tr>
<tr>
<td>17</td>
<td>specialized secondary school (after senior middle school) (graduate)</td>
</tr>
<tr>
<td>18</td>
<td>specialized secondary school (after senior middle school) (leave before graduate)</td>
</tr>
<tr>
<td>19</td>
<td>polytechnic college (graduate)</td>
</tr>
<tr>
<td>20</td>
<td>polytechnic college (leave before graduate)</td>
</tr>
<tr>
<td>21</td>
<td>TV university/correspondence course/long-distance education (graduate)</td>
</tr>
<tr>
<td>22</td>
<td>TV university/correspondence course/long-distance education (leave before graduate)</td>
</tr>
<tr>
<td>23</td>
<td>undergraduate (graduate)</td>
</tr>
<tr>
<td>24</td>
<td>undergraduate (leave before graduate)</td>
</tr>
<tr>
<td>25</td>
<td>Postgraduate (graduate)</td>
</tr>
<tr>
<td>26</td>
<td>Postgraduate (leave before graduate)</td>
</tr>
<tr>
<td>27</td>
<td>PhD (graduate)</td>
</tr>
<tr>
<td>28</td>
<td>PhD (leave before graduate)</td>
</tr>
</tbody>
</table>

Ordered probit category 1: Elementary (1 to 6 years of schooling): No 1 to No 6
Ordered probit category 2: Junior Middle (7 to 9 years of schooling): No 7 to No 8
Ordered probit category 3: Senior Middle (10 to 12 years of schooling): No 9 to No 18
Ordered probit category 4: High edu (above 13 years of schooling): No 19 to No 28
References


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